Chapter 19: Optimization and Equilibrium Combined

A Market for Pollution Permits

Taking Stock

You have learned to use pencil and paper and Excel to solve for the equilibrium solution of a system by imposing the equilibrium condition(s) on the structural equations and by generating various types of graphs (including the phase diagram) and tables. In addition, you have learned various ways of describing the equilibration process. You have applied these methods to a profit equilibrium model, a simple Keynesian macroeconomic model, and a rate of return model.

Today we will look at one more example of an equilibrium system: a market for government permits to emit a certain amount of pollution. This example utilizes optimization at the individual firm level, then brings the individual optimizing firms together in an equilibrium system. Thus, it enables us to review both optimization and equilibrium in the context of an interesting, contemporary problem.

The Future

In the next chapter, we will see how it is possible to look at the equilibrium system from the perspective of an overall, social optimization problem. We will compare the system-wide equilibrium and socially optimal solutions and reach a crucial, some might say stunning, result.

Introduction

THE PROBLEM—TOO MUCH LEAD POLLUTION

Lead has been used in a variety of industrial processes and products, especially gasoline and paint. Unfortunately, lead has been proven to have negative effects on the environment and human beings. Hundreds of thousands of people in the United States, for example, have unacceptably high levels of lead in their bodies. In children, overexposure to lead causes various neurological problems, while, for adults, high blood pressure is a primary concern.

As the evidence mounted that lead pollution was extremely harmful, the United States government began exploring alternative ways to reduce lead emissions from the use of gasoline. The government wanted to restrict the overall, industry-wide amount of such pollution. In the case of lead pollution, instead of following the conventional regulatory approach, the government implemented a novel strategy.
CONVENTIONAL REGULATION

In the past, the federal government has usually responded to pollution problems by setting a limit on how much pollution firms can generate. The primary virtue of this approach is that it is simple and understandable.

Let's suppose there are five firms, each polluting to the tune of 10 tons of gunk per month, and it is decided that the resulting damage from 50 tons of pollution to the environment is unacceptable. Pollution cannot be immediately eliminated, for this would force the shutdown of the firms and we would be unable to generate any output at all. Suppose it is decided that a target of 30 tons of pollution is desired. A conventional regulatory scheme, sometimes called "command and control" in the environmental economics literature, would allow each firm to emit, at most, 6 tons of pollution.

We hope you agree that it doesn't get much simpler than this. Given an agreed upon pollution limit, PL, the government takes the total amount of pollution allowed and divides by the number of firms and decrees, "Each firm is allowed to pollute up to PL/# FIRMS." They enforce the limit by on-site inspections and other auditing procedures. If 30 tons per month is the total pollution target (PL=30) and there are 5 polluting firms in the industry, Conventional Regulation says that each firm is allowed to pollute 6 (=30/5) tons per month.

MARKETABLE PERMITS

Recently, however, the Environmental Protection Agency (EPA) has turned from conventional regulation to market-based schemes for controlling pollution. The EPA gives each firm a certain number of "permits to pollute" and allows firms to trade those "permits to pollute" if they so desire. If a firm has 7 permits, it can pollute 7 units; no permits means it is not allowed to pollute at all. An active market for these permits, as firms buy and sell the rights to pollute, results in an allocation of the burden of reducing pollution that, surprisingly, improves upon the Conventional Regulation approach.

It all began in 1982, when government regulators and firms in the refining industry agreed that lead in gasoline posed a major hazard to the environment, so much so that its use in refining should be phased out. (Lead in gasoline ends up in the exhaust gases of cars and trucks.) The amount of lead in gasoline could be very easily monitored by the government, so compliance with the law was quite likely. The government decided that rather than put strict limits on each individual firm's lead use, it would issue permits to refiners to use lead in the production of gasoline which the refineries could trade among each other. Between 1982 and 1987, the total number of permits supplied each year declined as lead in gasoline was phased out. Today, refiners are not allowed to use any lead in the production of gasoline so that all gasoline is lead-free.

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1This means that unleaded gas is not gas which has had lead taken out, but gas which never had lead put in!
Questions to be Answered:

1) If Conventional Regulation is so simple and understandable, why did the EPA abandon it in the case of lead in the 1980s? What exactly is wrong with the Conventional Regulation approach that has each firm equally share in the burden of reducing pollution?

2) How does the market-based, permit scheme actually work?

We will answer both of these questions by working with a concrete example problem.

Reducing Lead Pollution in a Single Firm—A Simple Example

If the government decides that firms should cut back on lead in gasoline, how much does it cost an individual firm to comply with the regulation?

For simplicity, we’ll assume that each firm would use (and, thus, emit) 10 tons of lead per month if lead use was not restricted by the government.

If the firm can use as much lead as it wants (10 tons per month), then it does not have to install any new equipment or invest in any new technologies to refine petroleum without lead; hence, the cost of lead reduction is zero. As each firm is forced to cut back on lead use (from 10 to 9 to 8 tons/month, etc.), its cost of reducing lead emissions rises above zero. The first reductions in lead use are relatively inexpensive, but the additional costs of further reductions in lead use rise, so that eliminating the last few tons of lead is very expensive.

We capture this basic fact of increasing costs of additional pollution reduction with a simple model. Each firm faces a different specific cost schedule, but the general form of the cost function looks like this:

\[ TC(L) = \begin{cases} 
\frac{a}{2} L(10 - L)^2 & \text{if } L \neq 10 \\
0 & \text{if } L > 10 
\end{cases} \]

where: \( L \) = tons of lead used and emitted per month

Thus, \( 10 - L \) = amount of lead reduction

\( L \) must be equal to or greater than zero

\( TC(L) \) = total cost of lead reduction in dollars

\( a \) = an exogenous parameter \( (a > 0) \) that varies from one firm to another
To understand why the TC(L) functional form offered above might make sense, let’s consider the simple case where the value of a is 1. Below we present a table of the total costs of lead reduction as a function of L and a picture of those costs.

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Notice from the picture that the costs of reduction are decreasing in L, the number of tons per month of lead that the firm emits. This should make sense: the higher the lead emissions, the lower the reduction and the lower the cost of reduction.

It is a difficult relationship to grasp because it seems backwards. The idea is that less L increases costs because using lead lowers production costs—if you don’t use lead, you must use more expensive alternatives to refine the petroleum into gasoline.

For example, when L is 10, (10 - L) is 0, and the firm must undertake no lead reduction efforts; so total cost of pollution reduction is zero. When L = 0, on the other hand, the required reduction is 10 tons of lead/month, and TC(L) = a (10-L)^2 = (1)(10-0)^2 = $100 per month. The firm is forced to abandon the use of lead altogether and this is very expensive.
Reducing Lead Pollution in a Five Firm Industry

Now let's look at the case where there are five firms, or refineries, producing gasoline. We know that the five firms would use a total of 50 tons of lead per month in gasoline if they were not regulated. They would spend nothing on lead emissions reduction and they would cause great harm to the environment.

Suppose that the government decides to set a 30 ton/month pollution limit on the total amount of lead emissions this year. If firms are treated equally, then the PL = 30 ton/month pollution limit translates into a 6 ton/month limit for each of the 5 refineries. This is called Conventional Regulation.

Although each firm faces the same limit, different firms have different costs of reducing emissions. Some firms have newer refineries that can easily do without lead while other firms utilize older plants that are more costly to adapt to lead-less refining.

Let's suppose we knew the costs of lead reduction faced by the five firms:

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Notice how Firm 1 has the lowest costs of pollution reduction. In one month, it can go from using 10 units of lead to 9 for only $2.00 and it can get down to 5 for only $50.00. Firm 5, on the other hand, must have the oldest, creakiest plant around!

At 6/tons per month per firm, Conventional Regulation comes in at $240 per month (the sum of $32 + $40 + $48 + $56 + $64). The table above makes clear that if Firm 5 were allowed to use and emit 7 tons of lead each month, it would lower its pollution reduction costs by $28 (from $64 to $36). The industry could "pay for that" by forcing the new, more adaptable refinery of Firm 1 to use only 5 tons of lead, which would increase its costs by $18 (from $32 to $50). Thus, the SAME total amount of lead emissions of 30 tons/month could be achieved $10/month more cheaply!
The first of our two questions was:

1) If Conventional Regulation is so simple and understandable, why did the EPA abandon it in the case of lead in the 1980s? What exactly is wrong with the Conventional Regulation approach that has each firm equally share in the burden of reducing pollution?

We now have a clear answer:

Conventional Regulation reaches the target amount of pollution, but it fails to do so in the cheapest way possible—it is inefficient.

The important question is not whether or not Conventional Regulation achieves the target of 30 total tons of lead per month—it, of course, does.

In fact, Conventional Regulation can achieve ANY target. The important point is that there is a cost of pollution reduction associated with achieving the desired PL target via Conventional Regulation.

To make this crucial point clear, let’s turn to Microsoft Excel.

Getting Started on C19Lab.xls
Open the Microsoft Excel file C19Lab.xls and read the Introduction and ConvReg sheets. After you have drawn the graph in the Q&A sheet, return here.

The graph you just drew of cost of pollution reduction via Conventional Regulation for given pollution reduction targets is a clear reminder of the fact that pollution reduction is not free—there is an associated cost of pollution reduction for a given PL target.

To repeat the point made above:

“The important question is not whether or not Conventional Regulation achieves the target of 30 total tons of lead per month—it, of course, does.

The important question is, At what total cost does Conventional Regulation achieve the target of 30 tons of lead per month? In other words, economists ask, "Can the same result, PL = 30, be achieved more cheaply?" The answer is, "Yes!" It is this answer that has led to the market-based pollution permits scheme.
A key contribution of economists to the debate on pollution control has been their argument that Conventional Regulation is an overly expensive way to achieve desired goals of pollution reduction.

Economists convinced the EPA that, in the case of lead emissions, treating everyone the same was a bad idea. A better way to proceed would take into account the ability of firms to reduce pollution—those firms who could easily, cheaply reduce pollution should use and emit less lead than firms who could not.

A KEY POINT:
Instead of decreeing that all firms should be treated equally, the EPA looked for a way to take advantage of the lower costs of pollution reduction of certain firms.

Let's walk through an example so you can begin to see the logic of how the market for pollution permits would work.

Firm 1 would be willing to sell a permit to pollute one ton of lead/month to Firm 5 for anything over $18/month. Think about it. If Firm 1 sells a permit to pollute one ton of lead for say $25/month, it would make $7 each month. Sure, its pollution reduction costs would rise $18/month (from $32/month to $50/month) because Firm 1 would now be forced to reduce pollution to 5 tons of lead/month, but the sale of the permit would generate $25 every month.

Firm 5 would snap up the chance to pay $25/month to reduce the burden of its pollution compliance. If Firm 5 gets its hands on a permit that allows it to use and emit one more unit of lead, Firm 5 would be able to lower its pollution reduction costs by $28/month. Sure, it would have to pay $25/month, but Firm 5 would save $28/month leaving it $3/month ahead. Firm 5 would be willing to pay up to $28/month for a permit.

NOTE: Do take the time to look back at the table on page 5 to convince yourself that these numbers make sense. Use a pencil to do the calculations yourself.

While some students immediately see the logic in this example, others think, "These calculations are a big pain. Who would do this for $3/month?" Put yourself in charge of a major oil refinery, jack up the scale to, say, millions of dollars, and we think the answer is pretty plain—YOU would carefully figure out how much you'd gain by such an exchange with $3 million every month on the line!
Students sometimes think, "Yes, Firm 5 is better off with the permit at a price of $25/month than without the permit, but it would be even better off if it had gotten that permit at $20/month. Why did it buy the permit for $25?" That's a good point and one that economists are interested in studying. When two firms are involved, the final sale price depends on the bargaining abilities and positions of the two parties. But, often economists don't really care how the difference "maximum willingness to pay minus minimum willingness to accept" is divvied up between the agents. The most important point from the perspective of minimizing the cost of reaching the target goal is that the trade was made so that the pollution burden was reallocated in a way that lowered the total cost of pollution reduction.

Don't lose sight of what's happened here. We successfully described how and why Firm 1 would be more than happy to trade a permit with an equally willing Firm 5. They each did it because they ended up better off. In the process, they managed to lower the total cost of reaching the 30 ton per month lead target. This is crucial. Right now the allocation looks like this:

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We've reduced the total cost of meeting the 30 ton/month lead target from $240 to $230. Of course, there's room for more mutually advantageous bargaining here! But instead of slowing grinding our way through each trade, we'll use Microsoft Excel to learn more about how the market-based scheme works.

We'll see how each firm decides whether and how much to buy or sell and how the resulting equilibrium system that is the market for permits settles down and establishes an allocation of pollution burden that meets PL=30 at a lower cost than Conventional Regulation. Many have commented on the surprising ability of a seemingly uncoordinated group of individually optimizing agents to interact in such a way that they establish a rational, logical solution to a difficult allocation problem.
The Marketable Permits Scheme

One alternative to Conventional Regulation, and the one adopted by the EPA in the lead emissions case, is to set up a pollution permit market. This is the method of achieving pollution reduction that is endorsed by most economists. The explanation of how it works forms the subject of the remainder of this chapter.

Before we turn to individual firm demand curves for permits and the Excel workbook that shows how the equilibrium system operates, let’s make sure we understand some rudimentary concepts about permits and outline the crucial ideas we will be confronting.

Lead Permit Basics:

A pollution permit gives the lead refinery the legal right to use and emit a certain amount of lead over a certain period of time. Firms can use and emit only as many tons of lead as they have permits. Thus, if the government wants to reduce the amount of lead pollution, it need only issue fewer permits. This is, in fact, what the government did, as it phased out lead in gasoline.

Firms can trade permits freely at any price. Once a firm sells a permit, it must reduce pollution emissions by one unit; the buyer of a permit now holds the right to increase pollution by one unit.

If a firm has 5 permits, and it decides the optimal number of permits is 3, it will be a seller of permits. On the other hand, if it holds 2 permits, it will be a buyer of permits. Thus, a firm can be either a buyer or a seller of permits. Its position in the market depends on the price of permits and its pollution reduction technology. At high prices, the firm might be a seller, while at low prices it might switch and be a buyer!

Crucial Ideas:

We have seen that Conventional Regulation, although simple, is inefficient—it fails to minimize the cost of reaching a pollution target. We will show that a market-based permit scheme can reach the pollution target at a lower cost than an across the board pollution limit.

A second task will be to discuss not only the equilibrium solution, but also the equilibration process. We will explore the phase diagram that describes the equilibration process in the market for permits.

A final task will be to examine the comparative statics of our model. How, according to our model, would changes in the number of permits allowed by the government, as it phased out lead in gasoline, affect the equilibrium price of the permits? Our model will generate a clear prediction about the time path of prices in the lead permit market which we can actually check against the facts.
Individual Firm Demand for Permits

Before we can study the equilibrium system, we must determine the behavior of the interacting agents. In the pollution permits market, the interacting agents are the individual firms. We must determine how they decide how many permits to buy or sell at a particular price.

This boils down to another application of optimization theory. We can apply the recipe from the application of the Economic Approach to optimization problems.

Step 1: Setting Up the Problem

A) GOAL

The firm wishes to choose the amount of lead use that minimizes its total costs of pollution reduction keeping in mind the fact that it must have a pollution permit for every ton of lead used.

Thus, total costs of pollution reduction come from two places: (1) the cost of pollution reduction by using lead alternatives and new technology (which we've called TC(L)), and (2) the cost of buying permits to avoid having to reduce pollution (Price*L). This is captured by the following objective function:

\[
\text{minimize} \quad \text{Firm's Total Costs of Pollution Reduction} = \text{Own Costs of Reduction} + \text{Permit Costs}
\]

\[
\text{minimize} \quad \text{Firm's Total Costs of Pollution Reduction} = a \cdot (10 - L)^2 + \text{Price} \cdot L
\]

The firm's own costs of pollution reduction, the \( a \cdot (10-L)^2 \) part, seems to be saying, "Set \( L = 10 \) so that \( 10 - 10 \) equals zero," i.e., use a bunch of lead and pollute like crazy. But hang on a second, that choice of \( L \) might be very expensive (depending on the price of the permit, Price) because the firm would have to buy 10 permits in order to use 10 tons of lead. The firm has to optimize—that is, choose the amount of \( L \) that minimizes its Total Costs of Pollution Reduction.

B) ENDOGENOUS VARIABLES

The firm gets to choose the amount of lead use and emissions in tons per month. When it makes this choice, it is also deciding on the number of permits to buy because you need a permit for each ton of lead pollution.

So, \( L \) stands for both the amount of lead to be used in producing gasoline and the number of permits the firm will buy.
C) EXOGENOUS VARIABLES

The exogenous variables are the price of the permit, Price, and its own ability to reduce pollution which is given by the function:

\[ TC(L) = \begin{cases} 
  a(10 - L)^2 & \text{if } L \neq 10 \\
  0 & \text{if } L > 10 
\end{cases} \]

So “a” and the exponent “2” and the functional form itself of the TC(L) function are all exogenous to the firm.

Step 2: Find the Initial Solution

We have a variety of alternatives here, from tables and graphs to Excel’s Solver to calculus. We will present the solution strategy that we called the Method of Marginalism via Calculus. As you recall, this requires 5 steps:

**STEP (1)** Set up the problem mathematically; that is, write out the objective function, indicate the choice variable, and indicate what the agent wishes to do (i.e., MAX or MIN).

Minimize Firm’s Total Costs of Pollution Reduction

\[
\text{minimize } \text{Firm’s Total Costs of Pollution Reduction} = a (10 - L)^2 + \text{Price} \cdot L
\]

**STEP (2)** Take the derivative of the objective function with respect to the choice variable.

\[
\frac{d}{dL} \text{Firm’s Total Costs of Pollution Reduction} = -2a(10 - L) + \text{Price}
\]

**STEP (3)** Set the derivative of the objective function equal to zero. The value of the endogenous variable that satisfies this condition (called a “first-order condition” by mathematicians) is unique; we indicate this by sticking a star or asterisk (*) on the choice variable.

This is a crucial point. A choice variable, say \( X \), can be set by the agent to many different values. One (and only one) of these values is the optimal (or best or efficient) solution for the problem at hand. This special value, \( X^* \), is denoted by the asterisk convention.

\[
-2a(10 - L^*) + \text{Price} = 0
\]
STEP (4) Solve for the optimal value of the endogenous variable; that is, rearrange the equation (or first-order condition) in Step 3 so that you have X* by itself on the left-hand side and only exogenous variables on the right. This is called a reduced form: it tells you the optimal value of the endogenous variable for any set of values for the exogenous variables.

\[
2a(10 - L^*) = \text{Price}
\]

\[
(10 - L^*) = \frac{\text{Price}}{2a}
\]

\[
L^* = 10 - \frac{\text{Price}}{2a}
\]

STEP (5) Plug the optimal value back into the objective function. This will give you what is called the maximum value function. You can think of it as a sort of reduced form of the objective function: it gives the maximum value (or minimum value, in the case of, for example, the lifeguard problem) of the objective function as a function of the parameters of the problem.

Firm's Total Costs of Pollution Reduction* = \(a(10 - \left(10 - \frac{\text{Price}}{2a}\right)^2 + \text{Price} \left(10 - \frac{\text{Price}}{2a}\right)\)

The reduced form L* tells us, given "a" (the pollution reduction technology available to the firm) and the Price of pollution permits, how many tons of lead (and that same number of permits) a firm will want to use.

Firm 1 (on page 5) is a newer firm with an a=2. If Permits were $10 each, Firm 1 would want to set

\[
L_1^* = 10 - \frac{\text{Price}}{2a} \bigg|_{a=2, \text{Price} = 10} = 7.5
\]

and, thus, it would minimize its total costs of pollution reduction at $187.50 by using 7.5 tons of lead which requires buying 7.5 permits.

If it had 6 permits already, assuming the government had taken the 30 tons of lead reduction target and divided by 5 firms so that each firm had 6 permits, Firm 1 would be a buyer of 1.5 permits (assuming for the sake of argument that permits are infinitely divisible).

On the other hand, if the price of permits was higher, say $20, then Firm 1 would choose to use and emit 5 tons of lead (since \(10 - (20/4) = 5\)) and it would be a seller of 1 permit at that price.
The number of permits that firm wants to buy or sell is called the excess demand for permits. If positive, excess demand means the firm wants to buy permits; while excess demand less than zero means the firm wants to sell permits.

Once we have the reduced form $L^*$, finding the excess demand for permits is easy: merely subtract the number of permits the firm already holds.

Instead of subtracting 6, a more general formulation would be:

$$\text{Excess Demand for Permits}^1 = 10 - \frac{\text{Price}}{2a} - \frac{PL}{5}$$

For any values of $PL$, Price (of Permits) and "a," we can immediately determine the optimizing response of the firm in terms of the number of permits it wants to buy or sell. What remains to do now is to put the five optimizing firms together and let them trade!

Continuing with C19Lab.xls

Return to the Microsoft Excel file C19Lab.xls and read the IndFirm sheet. After you have completed the text box in Screen 2 of the Q&A sheet, return here.

The IndFirm sheet makes clear that firms are willing to be buyers or sellers of permits—it all depends upon the price of the permit and the firm’s particular “a” parameter that reveals its ability to reduce pollution on its own. Firms with low “a” (meaning that they have newer refineries that can cheaply avoid using lead) become sellers before firms with high “a” (for whom avoid lead use is more expensive). That’s something worth remembering.

You should also remember these other things that you’ve learned thus far:

- Conventional Regulation is a means of achieving a pollution reduction target by setting an equal maximum limit of pollution on each firm.

- In the lead pollution case, the EPA abandoned Conventional Regulation because it was clear that it was not the least cost method of achieving the target.

- The EPA adopted a market-based scheme in which firms actively bought and sold permits to pollute.

- Firms decide whether or not to buy or sell and how much to buy or sell by solving an optimization problem.

All that remains is to figure out the second question:

2) How does the market-based scheme actually work?
Understanding How the Market for Permits Works

Having used OPTIMIZATION to explore how individual firms decide how much to pollute and how many permits they want to buy at any given price, we now turn our attention to the MARKET for pollution permits.

The individual firm accepts the price as given (exogenous) and adjusts the choice variable, lead use and emissions (which is identically equal to the number of pollution permits), to its optimal value. The equilibrium viewpoint, on the other hand, ENDOGENIZES the price. In other words, price, from the perspective of the market as a whole, is determined by the interaction of the buyers and sellers in the market.

“Is it possible for price to be exogenous to the firm and endogenous to the system?” The answer is a resounding “YES!” The exogeneity or endogeneity of a variable is not something intrinsic to it, but depends on the perspective of analysis. Price can be exogenous to a competitive firm, but endogenous to a monopolist. It all depends on the framing of the problem.

Optimization does not disappear, because we view the actions of the individual firms as based on optimization, but it takes a back seat as we focus on how the system grinds out an equilibrium price that has the property that buyers' and sellers' plans are fulfilled.

It is important to keep straight where you are in the problem. We have completed the analysis of the individual optimizing firm and are now turning to an analysis of the equilibrium system as a whole.

I) Setting up the Problem

A Discussion of the Set Up:

A crucial part of the market for pollution permit analysis is the concept of market demand, or aggregate excess demand. This is just the sum of all individual firms' excess demands. The equilibrium condition is that the price must make market excess demand exactly equal to zero. Any price that does not do this is a disequilibrium price and means that the desires of buyers and sellers to not match. At the equilibrium price, the total number of permits offered for sale at that price is just equal to the total number of permits that other firms want to buy at that price.

The market for permits has price as an endogenous variable that bounces around as firms say whether they want to buy or sell and how much at any given price. The sheet IndFirm showed clearly that as price rises, firms want to sell more permits and previous buyers switch to become sellers. In the market for permits, at a low price, there are more buyers than sellers and the price is pushed up. On the other hand, at a high price, there are more sellers than buyers and the price is pushed down.
A More Formal Set Up:

Now we are ready to set up the model formally:

Endogenous Variables:
1) Price of permits

2) Amount of lead emitted by each firm (which equals the number of permits each firm ends up with) which is determined by the individual firm's optimizing given the system's equilibrium price.

Exogenous Variables:
1) "a" (each firm's cost of pollution reduction parameter)
2) PL (pollution limit or total emissions allowed by the government)

Structural Equations:
Five Individual Firm Excess Demand Functions one for each of the five firms):
[Excess Demand (ED) = Gross Demand - Initial Endowment of Permits]

\[
\begin{align*}
ED_1^* &= 10 - \frac{Price}{2a_1} - \frac{PL}{5} \\
ED_2^* &= 10 - \frac{Price}{2a_2} - \frac{PL}{5} \\
ED_3^* &= 10 - \frac{Price}{2a_3} - \frac{PL}{5} \\
ED_4^* &= 10 - \frac{Price}{2a_4} - \frac{PL}{5} \\
ED_5^* &= 10 - \frac{Price}{2a_5} - \frac{PL}{5}
\end{align*}
\]

Note how "a" and "PL" are exogenous and so, given Price, the firm can immediately say whether they want to buy or sell and how much. Note also that these equations come from the solution to each firm's optimization problem.

Aggregate (or Market) Excess Demand:
Aggregate ED = ED1 + ED2 + ED3 + ED4 + ED5

Equilibrium Condition: Aggregate ED = 0
II) Finding the Equilibrium Solution

To solve for \( P_e \) (and the resulting Lead use for each firm), we have several alternatives at our disposal, including: a pencil and paper or "by hand" strategy or various computer methods (table, graph, or Excel's Solver). We will demonstrate the paper and pencil method here.

We must find that level of price which satisfies the equilibrium condition. The notation is crucial—\( P_e \) is one particular price; one point among the infinite possible values the variable \( P \) could be. In verbal terms, we are searching for the price where the aggregate amount of permits bought is exactly equal to the aggregate amount sold so that aggregate excess demand is zero.

**A Pencil and Paper (or “by hand”) Solution:**

To solve an equilibrium problem with pencil and paper, you simply follow a recipe.

**STEP (1): Write the structural equation(s) and equilibrium condition(s)**

For this problem, we have:

**Structural Equations:**

\[
\begin{align*}
ED_1^* &= 10 - \frac{\text{Price}}{2 \, a_1} - \frac{\text{PL}}{5} \\
ED_2^* &= 10 - \frac{\text{Price}}{2 \, a_2} - \frac{\text{PL}}{5} \\
ED_3^* &= 10 - \frac{\text{Price}}{2 \, a_3} - \frac{\text{PL}}{5} \\
ED_4^* &= 10 - \frac{\text{Price}}{2 \, a_4} - \frac{\text{PL}}{5} \\
ED_5^* &= 10 - \frac{\text{Price}}{2 \, a_5} - \frac{\text{PL}}{5} \\
\text{AED} &= ED_1^* + ED_2^* + ED_3^* + ED_4^* + ED_5^*
\end{align*}
\]

**Equilibrium Condition:**

\[ \text{AED} = 0 \]

Price is the key endogenous variable. It will continue to change as long as \( \text{AED} \neq 0 \).
STEP (2): Force the structural equations to obey the equilibrium conditions.

We do this by writing:

\[
\begin{align*}
ED_1^* &= 10 - \frac{Price_e}{2a_1} - \frac{PL}{5} \\
ED_2^* &= 10 - \frac{Price_e}{2a_2} - \frac{PL}{5} \\
ED_3^* &= 10 - \frac{Price_e}{2a_3} - \frac{PL}{5} \\
ED_4^* &= 10 - \frac{Price_e}{2a_4} - \frac{PL}{5} \\
ED_5^* &= 10 - \frac{Price_e}{2a_5} - \frac{PL}{5} \\
ED_1^* + ED_2^* + ED_3^* + ED_4^* + ED_5^* &= 0
\end{align*}
\]

Notice that we attached an "e" subscript to Price when we made the structural equations equal the equilibrium condition. This is just like putting an asterisk (*) on the endogenous variable when we set the first order conditions equal to zero. The subscript “e” is immediately attached to “Price” because we are saying, the moment we write the equation, that Price_e represents the equilibrium value of price—that is, the value of price that makes the sum of the excess demands equal to zero.

As before, we have, strictly speaking, found the answer. We rewrite the equation, however, for ease of human understanding, as a reduced form expression.

STEP (3): Solve for the equilibrium value of the endogenous variable; that is, rearrange the equations in Step 2 so that you have Price_e by itself on the left-hand side and only exogenous variables on the right. This is called a reduced form: it tells you the equilibrium value of the endogenous variable for any set of values for the exogenous variables.

The math is cumbersome, but easy. You simply force the ED of each firm to meet the equilibrium condition, that the sum of the individual excess demands is zero. The six equations have only six unknowns (the five EDs and Price). Substituting the EDs into the equilibrium condition yields:

\[
\begin{align*}
10 - \frac{Price_e}{2a_1} - \frac{PL}{5} + 10 - \frac{Price_e}{2a_2} - \frac{PL}{5} + 10 - \frac{Price_e}{2a_3} - \frac{PL}{5} + 10 - \frac{Price_e}{2a_4} - \frac{PL}{5} + 10 - \frac{Price_e}{2a_5} - \frac{PL}{5} &= 0
\end{align*}
\]
This can rearranged as:

\[
50 - PL = \frac{\text{Price}_e}{2a_1} + \frac{\text{Price}_e}{2a_2} + \frac{\text{Price}_e}{2a_3} + \frac{\text{Price}_e}{2a_4} + \frac{\text{Price}_e}{2a_5}
\]

Solving for \( \text{Price}_e \) yields:

\[
\text{Price}_e = \frac{50 - PL}{\frac{1}{2a_1} + \frac{1}{2a_2} + \frac{1}{2a_3} + \frac{1}{2a_4} + \frac{1}{2a_5}}
\]

The above expression is the "reduced form" of the system of equations that compose this model because it expresses the equilibrium value of the endogenous variable as a function of exogenous variables alone. The expression can be evaluated at any combination of the exogenous variables in order to determine \( \text{Price}_e \).

For example, given \( PL = 30 \) and the five firms we were working with before \( (a_1 = 2, a_2 = 2.5, a_3 = 3, a_4 = 3.5, a_5 = 4) \), the equilibrium price is

\[
\]

\[
\text{Price}_e = \$22.61
\]

At this price of $22.61, the amount firms want to sell is exactly equal to the amount firms want to buy so that the market excess demand is exactly zero.

Another way to find the equilibrium price is to use Excel’s Solver.

---

**Continuing with C19Lab.xls**

Return to the Microsoft Excel file C19Lab.xls and read the Eq Solution sheet. After you have completed your text box and filled in the table in Screen 3 of the Q&A sheet, return here.

Welcome back from the Eq Solution sheet.

Price = $10 cannot be equilibrium because there are too many buyers and not enough sellers at that price. What will happen next? How will the price adjust? These questions are questions about the equilibration process.
**Understanding the Equilibration Process**

We have found the equilibrium solution, \( P_{eq} = \$22.61 \) via pencil and paper and Excel's Solver (and, as usual, they give the same answer!). The next step in analyzing an equilibrium system concerns the equilibration process itself.

As you know, phase diagrams have a powerful consistency that can be exploited to quickly answer the three kinds of questions about the equilibration process:

- Is the equilibrium stable?
- How is it reached?
- How fast is it reached?

We will look at a phase diagram in just a moment.

**A Discussion of the Equilibration Process:**

As we have seen, given the current price, each firm computes its excess demand by solving a cost minimization problem. The solution to the firm's minimization problem is the firm's gross demand. Excess demand is gross demand minus the firm's initial endowment of permits, \( PL/5 \). The sum of the individual excess demands is the market excess demand.

There's a familiar story one can tell about the typical equilibrating process in a market.

- If there is positive excess demand, this means that buyers are not able to buy all they want to buy at the current price. Thus, buyers compete with one another by bidding up the price.

- On the other hand, if excess demand is negative, sellers are not able to sell all they want to sell at the current price. Then sellers compete with one another by cutting prices in order to attract more buyers.

The implications of the familiar story are:

- If market excess demand at a given price is positive, there will be pressure to push the price up.
- If market excess demand is negative at a given price, there will be pressure to push the price down.
- If market excess demand is zero at a given price, then there will be no pressure to change the price. The price will remain at rest—this is the equilibrium price.
**The Equations of Motions of the Equilibrium System:**

From the above discussion, we can specify the equations of motion:

\[ P(t+1) = P(t) + \Delta P(t+1) \]

where \( \Delta P(t+1) = \alpha \times \text{MarketED}(P(t)) \)

Basically, the equation above says that the change (\( \Delta \)) in price in the next period is some proportion (\( \alpha \)) of the market excess demand in the current period. \( \alpha \) is some exogenous variable whose value depends on the system you're looking at. Different values of \( \alpha \) will generate different kinds of equilibration.

That's the story on a general level. Now, let's slowly and carefully try to figure out more precisely what's going on by walking through some numbers.

Suppose the initial level of the value of price is \( P = \$10 \). Then, our structural equations tell us that market excess demand is about 11.15 permits. Quantity demanded is greater than quantity supplied—the price will be pushed up. By how much does price rise? This is crucial to a specific description of the equilibration process.

Suppose that, because of the speed with which firms respond to the depleted inventories, output increases by 1.5 times the market excess demand so that the value of \( \alpha \) is 1.5. According to our description of the equilibration process, the change in \( P \) in the next period will be

\[ \Delta P(t+1) = 1.5 \times 11.15 \approx \$16.73/\text{permit}, \]

so that the price next period will be \( \$26.73 \) since that's equal to \( \$16.73 + \$10 \).

Let's see what happens the next period. With \( P = \$26.73 \), market ED is -3.65, and price falls by \$5.47 to \$21.26. And so on, until the value of output "settles down", i.e., reaches an equilibrium solution.

**More on \( \alpha \):**

The key to the equilibration process is \( \alpha \). Notice that we made \( \alpha > 0 \). Why? Because price rises when market ED is greater than zero; and price falls when market ED is less than zero. If the economic argument that price and market ED are directly related holds true, \( \alpha \) will be positive. The size and sign of the exogenous variable "\( \alpha \)" describes the speed and type of the equilibration process.

**More on Excel:**

If you are thinking that there's no way you are going to chug through repeated calculations of the sort found on this page, don't worry, YOU WON'T HAVE TO!
This type of mindless calculation is what computers are exceptionally good at!!!

Return to Excel now and go to the sheet called EQ PROCESS. It's time to put these lessons to work! Return here and go to the next page when you are done.

Welcome back from the Eq Process sheet.

Having finished studying the equilibration process, we can make a crucial point:

The Marketable Permits strategy beats the Conventional Regulation scheme!

Although involved in minimizing its own individual cost, each firm helped establish an allocation of pollution emission that is a lower cost alternative to Conventional Regulation. This rather remarkable result is captured in the table below:

### Pollution Permit Market Results

<table>
<thead>
<tr>
<th>Initial Number of permits</th>
<th>Number bought</th>
<th>Number sold</th>
<th>Equilibrium Price</th>
<th>Resulting Allocation</th>
<th>Comparing the Two Schemes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Firm 1</td>
<td>6</td>
<td>1.65</td>
<td>$22.61</td>
<td>4.35</td>
<td>$ 63.85 $ 32.00</td>
</tr>
<tr>
<td>Firm 2</td>
<td>6</td>
<td>0.52</td>
<td></td>
<td>5.48</td>
<td>$ 51.08 $ 40.00</td>
</tr>
<tr>
<td>Firm 3</td>
<td>6</td>
<td>0.23</td>
<td></td>
<td>6.23</td>
<td>$ 42.64 $ 48.00</td>
</tr>
<tr>
<td>Firm 4</td>
<td>6</td>
<td>0.77</td>
<td></td>
<td>6.77</td>
<td>$ 36.52 $ 56.00</td>
</tr>
<tr>
<td>Firm 5</td>
<td>6</td>
<td>1.17</td>
<td></td>
<td>7.17</td>
<td>$ 32.04 $ 64.00</td>
</tr>
</tbody>
</table>

**TOTAL** 30 2.17 2.17 30.00 $226.11 $240.00

The table above deserves your close attention.

Notice how Firm 1, the newer, low cost pollution avoider is doing exactly what we wanted—it uses only 4.35 tons of lead per month, the lowest in the industry. Firm 5, on the other hand, the wheezing, old refinery that finds it costly to avoid lead use, is the highest lead user of the five.
Under Conventional Regulation, 1 and 5 are treated the same (each uses 6 tons of lead) and this leads to higher cost of pollution reduction (a total of $240.00) than is necessary. The marketable permits scheme provides the same total amount of lead use at a lower cost than Conventional Regulation.

The remarkable result that marketable permits beats conventional regulation merits further discussion. In particular, you might be wondering why we have ignored the costs of buying the permits themselves. In fact, we haven't. When the market reaches its equilibrium solution, the quantity demanded equals the quantity supplied so added costs to permit buyers are exactly offset by reduced costs to permit sellers. Perhaps another table would help:

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Firm 3</th>
<th>Firm 4</th>
<th>Firm 5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value of “a”</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
<td>3.5</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

**DIRECT REGULATION**

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Firm 3</th>
<th>Firm 4</th>
<th>Firm 5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead Use</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>30</td>
</tr>
<tr>
<td>Firm Cost of Pollution Reduction</td>
<td>$32.00</td>
<td>$40.00</td>
<td>$48.00</td>
<td>$56.00</td>
<td>$64.00</td>
<td>$240.00</td>
</tr>
<tr>
<td>Total Cost of Pollution Reduction</td>
<td>$32.00</td>
<td>$40.00</td>
<td>$48.00</td>
<td>$56.00</td>
<td>$64.00</td>
<td>$240.00</td>
</tr>
</tbody>
</table>

**MARKETABLE PERMITS**

<table>
<thead>
<tr>
<th></th>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Firm 3</th>
<th>Firm 4</th>
<th>Firm 5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead Use</td>
<td>4.35</td>
<td>5.48</td>
<td>6.23</td>
<td>6.77</td>
<td>7.17</td>
<td>30</td>
</tr>
<tr>
<td>Firm Cost of Pollution Reduction</td>
<td>$26.54</td>
<td>$39.32</td>
<td>$47.84</td>
<td>$53.92</td>
<td>$58.49</td>
<td>$226.11</td>
</tr>
<tr>
<td>Total Cost of Pollution Reduction</td>
<td>$63.85</td>
<td>$51.08</td>
<td>$42.64</td>
<td>$36.52</td>
<td>$32.04</td>
<td>$226.11</td>
</tr>
</tbody>
</table>

Under Direct Regulation, there is no buying or selling of permits. Firms are simply told that they can use 6 tons of lead and, thus, they must each reduce their lead emissions by 4 tons. This costs Firm 1 $32.00 and Firm 5 $64.00. The total costs to the firms and to society for reducing lead use to 30 tons is $240.00.

Now, under the Marketable Permits scheme, something interesting happens. Look at the bottom two rows of the table. Firm 1 has a lower firm cost of pollution reduction, but a higher total cost of pollution reduction. How can that be? Because it pollutes only 4.35 tons of lead instead of 6 tons of lead, it has higher costs of pollution reduction in terms of resources spent on reducing pollution. But it is more than compensated for this because it SELLS PERMITS. Firm 1 makes $63.85 - $26.54 = $37.31 on its permit sales so its cost of pollution reduction is $37.31 = $26.54. It’s happier under the permit scheme than the direct regulation framework because its own costs are lower ($26.54 versus $32.00). A similar trade-off happens with Firm 5. Its own costs are lower under the
permits scheme ($58.49 versus $64.00) because it lowers its costs of pollution reduction by more than the costs of the permits it must buy. Firm 5’s $58.49 costs of pollution reduction is composed of $32.04 of resources devoted to reducing lead use to 7.17 tons plus \(1.17 \times 22.61 = 26.45\) for the permits it must buy. Firm 5 prefers this to the Direct Regulation scheme because its costs are lower.

The fact that the Marketable Permits scheme beats Direct Regulation is not an easy thing to understand. It is rather counter-intuitive and puzzling. Perhaps the secret to the puzzle lies in realizing that firms are in different situations. Some can reduce pollution easily while others may find it very expensive. If that’s true, then it might be cheaper to buy a permit to pollute than to reduce pollution. That firm will buy permits. On the other hand, the firm which can easily reduce pollution will sell permits because it will end up ahead since it takes in more money from the sale of permits than it loses by having to reduce pollution.

What the market does is make each trader, both buyer and seller, better off:

<table>
<thead>
<tr>
<th>Firm 1</th>
<th>Firm 2</th>
<th>Firm 3</th>
<th>Firm 4</th>
<th>Firm 5</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DIRECT REGULATION</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead Use</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Firm Cost of Pollution Reduction</td>
<td>$32.00</td>
<td>$40.00</td>
<td>$48.00</td>
<td>$56.00</td>
<td>$64.00</td>
</tr>
<tr>
<td><strong>MARKETABLE PERMITS</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lead Use</td>
<td>4.35</td>
<td>5.48</td>
<td>6.23</td>
<td>6.77</td>
<td>7.17</td>
</tr>
<tr>
<td>Firm Cost of Pollution Reduction</td>
<td>$26.54</td>
<td>$39.32</td>
<td>$47.84</td>
<td>$53.92</td>
<td>$58.49</td>
</tr>
<tr>
<td>Gain from Marketable Permits</td>
<td>$5.46</td>
<td>$0.68</td>
<td>$0.16</td>
<td>$2.08</td>
<td>$5.51</td>
</tr>
</tbody>
</table>

Every firm has lower costs with marketable permits and, therefore, ends up ahead! While each firm is zealously looking at its own bottom line and acting to minimize its costs of pollution reduction, something else is also going on—the firms are reallocating the 30 tons of lead in a way that lowers the total cost of meeting the 30 ton lead use standard. As the bottom row in the table above shows, each firm shares in the $13.89 savings generated by the marketable permits scheme. Economists since Adam Smith have expressed awe and amazement at the fact that trading can generate a better allocation than direct control.

Perhaps the best way to understand the Marketable Permits scheme is to separate out two distinct problems: the individual firm and overall lead allocation problems. (1) Each firm is happier under the permits scheme since its costs are lower AND (2) the resulting allocation of the 30 tons of lead use is lower than under Direct Regulation because firms like Firm 5 which find it very expensive to reduce pollution buy permits from those like Firm 1 which can cheaply and easily reduce pollution.
Having discussed the way in which the marketable permits scheme works as the Initial Solution to the equilibrium model, we can now turn to the last step of the Economic Approach, comparative statics.

**Comparative Statics**

The crucial comparative statics exercise in the marketable permits model is an exploration of how the endogenous variable equilibrium price responds to shocks in the federally mandated target level of total pollution emissions.

The EPA controls the total amount of lead pollution in the industry by deciding how many permits to issue each firm. With five firms and PL = 30, each firm started with six permits. The market reached an equilibrium solution at \( P_{eq} = \$22.61 \). But what if PL = 20 or 40? What would be the equilibrium price then?

To find out, let's use Excel's Solver and the Comparative Statics Wizard to track how the equilibrium price changes as PL changes.

Return to Excel now and go to the sheet called COMP STATICS.

When you are done with the COMP STATICS sheet, you have finished this lab. Thank you for your hard work.

In the next chapter, we will present an even more outlandish claim concerning the Permit Market.