Chapter 16: Equilibrium in a Macroeconomic Model

Introduction:

When famed British economist John Maynard Keynes published *The General Theory of Employment Interest and Money* in 1936, he was, as always, supremely confident. In a letter to George Bernard Shaw in 1935, he said that

> I believe myself to be a writing a book on economic theory which will largely revolutionize—not, I suppose, at once, but in the course of the next ten years—the way the world thinks about economic problems. . . . I can't expect you or anyone else to believe this at the present stage. But for myself I don't merely hope what I say, in my own mind, I am quite sure.[1]

There is no doubt that Keynes had the impact he thought he would have. Most macropolicy between the 1940s and the 1970s was guided by Keynesian principles and even now a large number of economists and policy makers view the economy through Keynesian lenses. The Simple Keynesian Model is, as its name suggests, the most basic model in the Keynesian family. Although highly abstract (even by the standards of macro models), the Simple Keynesian Model is helpful for its ability to highlight the fundamental equilibrating forces common to all Keynesian macro models.

We will use the Simple Keynesian Model to illustrate the notions of the equilibrium solution, the equilibration process, and the comparative statics properties that are common to all equilibrium systems. Although this is a completely different application from the Profit Equilibration Model, we will see the same logic and ideas repeated.

Organization:

After reviewing the most important assumptions of the model, we will analyze it with words and graphs. With an understanding of the logic behind the model, we will present an exposition of the Simple Keynesian Model as yet another application of the Economic Approach. We will then explore the **equilibration process** by discussing the phase diagram associated with the model. Finally, we will examine the comparative statics properties of the model.

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[1] For more on Keynes and his life start with Robert Heilbroner's *The Worldly Philosophers*, then try Roy Harrod's *The Life of John Maynard Keynes* (from which the above quotation is taken, p. 462).

Fundamental Assumptions:[2]

(1) The flow of output produced by an economy (GDP) in a given time period is **identically equal** to income (Y) generated. This identity is due to the fact that everything produced and sold in the economy results in a payment to the inputs that produced it in the form of rent, wages, interest, or dividends. Y (playing the dual role of output and income) is the endogenous variable in the model that measures the dollar flow during a specific time period.

(2) Output is demanded by three types of agents: consumers, firms, and the government.

(A) Individual consumers' demands for products can be aggregated and represented by a **consumption** function. Furthermore, consumption is completely determined by disposable income ($Y - t_0Y$) (where "t_0" is a given flat tax rate which is constant across all income levels). Then, we can write

$$C = c_0 + MPC(Y-t_0Y),$$

where c_0 is the intercept of the consumption function (sometimes called "autonomous consumption") and the MPC, the marginal propensity to consume, is the slope.

(B) Individual firms' demands for capital goods can be aggregated and represented by a **planned investment** function. We are going to assume that planned investment is determined exogenously; that is, the level of investment at any point in time is given and unaffected by changes in other variables. Then, we can write,

$$I = I_0.$$

The "0" subscript indicates some initial value of investment spending.

(C) Government demand for output is exogenously determined and can be represented by the government spending function:

$$G = G_0.$$

As above, the "0" subscript indicates some initial value of investment spending.

(3) The price level is constant, i.e., there is no inflation. Therefore, the nominal values of Y, C, I, and G are also their real values.

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2 We say "Fundamental Assumptions" to highlight the fact that we are not listing ALL of the assumptions that describe the Keynesian model. If we really got serious and listed all of the assumptions, the task would be quite time-consuming and not particularly useful for our purposes. Examples of unstated assumptions include: a closed economy, no capital depreciation, unchanging technology, among others. We will leave the full characterization to your macro professor and concentrate on the equilibration process.

The Model in Words:

Equilibrium (defined as a state in which there is **no tendency to change** or **a position of rest**) will be found when the desired amount of output demanded by all the agents in the economy exactly equals the amount produced in a given time period.

There are three classes of demanders or buyers of goods: consumers, firms, and the government. It is clear that consumers demand output so they can consume, but perhaps less obvious that firms also demand output so they can invest.

Investment means something specific in economics: it is the act of buying capital goods (tools, plant, and equipment). You might say, "I invested my money in the stock market," but that's *not* the economic definition of investment. Investment, in economics, represents the fact that firms, while sellers of the products they make, must also BUY things in order to produce. Investment, strictly defined, is those goods and services bought by firms in order to produce their output.

Clearly, the total desired amount of output demanded, or ***aggregate demand*** (AD), is the simple sum of the consumption function, investment function, and government spending—i.e., the sum of the demands of the three types of buyers.

At any level of income, aggregate demand may be (1) greater than, (2) less than or (3) equal to the actual amount of goods and services produced by the economy in the period (i.e., ***actual GDP***). However, only in the last case (AD = actual GDP) will the economy be in equilibrium. If AD does not equal actual GDP, the system has a tendency to change to a different level of output (and national income).

ASIDE: For purposes of measurement of economic flows, the period of time chosen is usually monthly, quarterly or annually. To simplify our discussion, we will focus on annual measures of income and output. GDP, then, is the market value of final goods and services produced by the economy in one year's time.

The Three Cases Considered:

(1) For a given level of income, if **AD is greater than actual GDP**, firms will be forced to meet demand by drawing down their inventories. This results in inventory stocks that are too low relative to the optimum amount of inventory, so in response firms will **produce more** (that is, increase Y) in the next period to get their inventories back up to where they want them to be (note that we are assuming some "optimal" level of inventories). Because there is a tendency to change, this cannot be the equilibrium level of income. Another way of saying this is that the change in inventories is the **signal** upon which firms base their **decision** of whether to increase or decrease production.

(2) Suppose at the new, higher level of output that **AD is now less than actual GDP**. Firms will be unable to sell all of their output, and as a result their inventories will rise as unsold products

accumulate. In an attempt to return inventories to their optimal level, firms will **produce less** (that is, decrease Y) in the next period. Once again, this level of output has a tendency to change—this eliminates it from consideration as the equilibrium level of output.

(3) In fact, under certain conditions (primarily, a well-behaved AD function), there is only one level of output that has no tendency to change—that is, the level of output where AD exactly equals actual GDP. Firms find that the output produced is exactly sold, with neither a depletion, nor an addition to optimal inventory levels. In response to this, they produce the **exact same** level of output next period—and every period after that as long as AD=actual GDP. For this reason, this level of output is called the ***equilibrium level of output (or national income)***—i.e., the level of output (or national income) at which there is no tendency to change.

Two points must be emphasized about our Simple Keynesian model of the economy:

POINT 1: The Keynesian model described above is completely *demand-driven*. Demanders always get what they want. Even when the sum of consumer, firm, and government demands is greater than what is produced in any given period, firms can always meet demand by "going to the back room" and selling output produced in previous periods—i.e., inventories. Furthermore, we are assuming that output in the next period can always be expanded if aggregate demand exceeds the current output level.

POINT 2: There is a crucial relationship between the amount produced and the amount consumers want to buy. This relationship is what drives the model to equilibrium. Remember, in this simple model, firms and the government want to buy the same amount no matter what their income level; consumers' demand, however, is a function of disposable income. The fact that disposable income changes when output changes (as more workers are hired or fired) is what moves the model to equilibrium.

If AD is greater than output, more output is produced next period by hiring more workers, which means that consumers have more income and will buy more. But the increase in demand will be less than the increase in output because they will save some of their extra income.

That's confusing, we know, but that's how it works. Macroeconomists often resort to graphs to help explain the workings of the Keynesian model. Let's try that strategy now.

The Model in Pictures:

The consumer, firm and government demands can be represented graphically. Although several graphical presentations are possible, Figure 1 below is the most common. It is known as the "Keynesian cross" because of the upward sloping AD and 45° lines. It is important to know that the intersection of these two lines yields the equilibrium level of national income, but it is much more important to know **why**.

In Figure 1, the slope of line C (the consumption function) is **constant** and **less than one** reflecting the assumed properties of the MPC and the tax rate. The lines I and G reflect the exogenously-determined investment and government demands for output. **AD is a simple sum of the three demands for each given level of income, Y.** Note that (1) AD is parallel to C because the other two demands have a zero slope and that (2) the distance between AD and C equals the sum of I and G. The slopes tell us how the components of demand change as income changes. A zero slope for I means that investment spending, for example, does not change as income changes. The fact that the slope of the consumption function is less than one means that a one-dollar increase in national income leads to less than a one dollar increase in consumption.

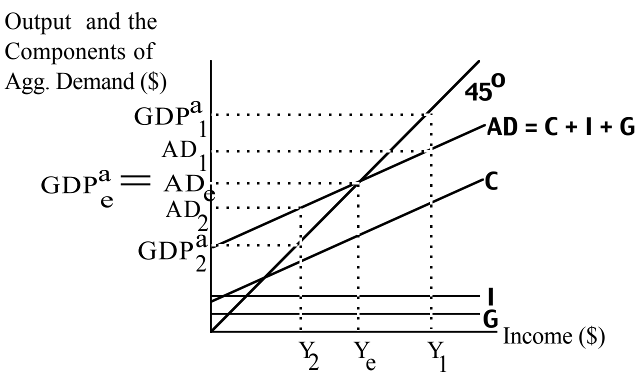

Figure 1: Equilibrium Income and Output via the Keynesian Cross

A common source of confusion concerns the units and variables on the axes. The y-axis is measured in dollars per year. C, I, G, AD, and actual GDP (GDP^a) can all be read off the y-axis. The x-axis is also measured in dollars per year. Actual GDP can be easily measured on the y-axis by use of the 45° line (where it can be compared to AD). Because of the first assumption, (see page 2) actual GDP equals Y. As an example of how to read the figure, when national income is Y_2, investment equals I_0, government spending equals G_0, actual output ($GDP^a{}_2$) equals national income (Y_2). Aggregate Demand equals AD_2. The vertical distance above the C line up to the AD line is equal to the sum of I_0 and G_0. Because C and AD are parallel, this vertical distance is always the same.

Equilibrium occurs at Y_e because this is the only level of output at which there is no tendency to change. The 45° degree line translates actual output on the horizontal axis to the vertical axis. **The 45° line allows us to compare a given level of actual GDP and AD on the vertical axis**. If they are not equal, we know from the discussion above that firms will adjust output; consequently, that level of output cannot be Y_e.

At Y_1, for example, actual GDP (GDP^a_1) is greater than AD (AD_1); we know firms will respond to increases in inventory by decreasing output. At Y_2, actual GDP (GDP^a_2) is less than AD (AD_2) and, thus, Y will increase as firms rebuild their diminished inventories. **Only at Y_e, where actual GDP (GDP^a_e) exactly equals AD (AD_e), is there no tendency to change—because inventories are unchanged.**

Before we turn to an exposition that utilizes mathematics to present the model, use the graph to reinforce your understanding of the crucial equilibrating role played by the consumption function. Since the slope is less than one, an additional dollar of income will lead to less than an additional dollar of AD. That's why, when AD is greater than output and firms respond by producing more, the resulting increase in income does not eat up the entire increase in output. Notice also how AD less than actual GDP shows up on the graph *to the left* of the equilibrium solution; while AD greater than actual GDP occurs *to the right* of the equilibrium solution. Only at the intersection of the AD and 45^o degree lines do we get the third case were AD=actual GDP and the system shows no tendency to change.

Having sketched out the Simple Keynesian Model in words and with the "Keynesian cross" graph, we now turn to a presentation that returns us to the Economic Approach as applied to Equilibrium models.

The Simple Keynesian Model as Another Application of the Economic Approach:

I) Setting Up the Problem

We will begin the construction of the Simple Keynesian Model by laying out the pieces we have to work with. We first separate all of the variables into two mutually exclusive categories called "endogenous" and "exogenous" variables. The endogenous variables are those that are determined by the forces in the system. In the Simple Keynesian Model, the crucial endogenous variable is the level of output (and income), Y. We should note that C and AD are also endogenously determined by the forces in the model.

The exogenous variables are those **fixed, given conditions that comprise the environment in which the system works.** The exogenous variables of primary importance in the Simple Keynesian model include:

- **the components of the consumption function (i.e., the intercept, the marginal propensity to consume, and the income tax rate)**
- **planned investment spending**
- **government spending**

The next step in building an equilibrium model is to write the "system" as a set of equations. There are four structural equations in the Simple Keynesian Model, representing the four demands for output and their sum:

$$C = c_0 + MPC(Y - t_0 Y)$$
$$I = I_0$$
$$G = G_0$$
$$AD = C + I + G$$

(Remember that the "0" subscripts are just a commonly used notation to indicate the initial value of the particular exogenous variable.)

The final piece to be laid out is the equilibrium condition:

$$Y = AD$$

That is, the equilibrium level of output will be that level that satisfies the condition that actual GDP ("Y," on the left hand side of the equilibrium condition) equals the sum of the demands of all the agents in the economy (denoted by the right hand side's "AD"). Notice that since national income (Y) equals actual GDP, we can use Y instead of GDP[a] to represent output. This is helpful, because AD is a function of Y in our model.

II) Finding the Equilibrium Solution

To solve for Y_e, we have several alternatives at our disposal, including: a pencil and paper or "by hand" strategy or various computer methods (table, graph, or Excel's Solver). We will demonstrate the paper and pencil method here.

We must find that level of output which satisfies the equilibrium condition. The notation is crucial here—Y_e is one particular level of output; one point among the infinite possible levels the variable Y could be. In verbal terms, we are searching for the level of output (and national income) where the amount produced is just exactly equal to the total amounts desired by the consumers, firms, and the government. In graphical terms, we are searching for the intersection point of AD and the 45^o line.

A Pencil and Paper (or "by hand") Solution:

To solve an equilibrium problem with pencil and paper, you simply follow a recipe (just like we did with optimization and with the Simple Profit Equilibration Model). The recipe for equilibrium models is to write out the necessary equations of the model, then solve for the equilibrium values of the endogenous variables.

STEP (1): Write the structural equation(s) and equilibrium condition(s)

For this problem, we have:

Structural Equations:

$$\mathbf{C = c_0 + MPC(Y - t_0 Y)}$$
$$\mathbf{I = I_0}$$
$$\mathbf{G = G_0}$$
$$\mathbf{AD = C + I + G}$$

and

Equilibrium Condition:

$$\mathbf{Y = AD}$$

Y is the key endogenous variable. It will continue to change as long as $Y \neq AD$.

STEP (2): Force the structural equations to obey the equilibrium conditions.

We do this by writing:

$$\mathbf{Y_e = c_0 + MPC(Y_e - t_0 Y_e) + I_0 + G_0}$$

Notice that we attached an "e" subscript to Y when we made the structural equation equal the equilibrium condition. This is just like putting an asterisk (*) on the endogenous variable when we set the first order conditions equal to zero. The subscript "e" is immediately attached to "Y" because we are saying, *the moment we write the equation*, that Y_e represents the equilibrium value of output—that is, the value of output that makes actual GDP equal to the sum of the demands.

As before, we have, strictly speaking, found the answer. We rewrite the equation, however, for ease of human understanding, as a reduced form expression.

STEP (3): **Solve** for the equilibrium value of the endogenous variable; that is, **rearrange** the equation in Step 2 so that you have Y_e by itself on the left-hand side and only exogenous variables on the right. This is called a **reduced form**: it tells you the equilibrium value of the endogenous variable for any set of values for the exogenous variables.

Y_e appears on both sides of the equation. **We want to solve for Y_e.** To do that, we can simply follow these steps:

(1) Factor Y_e out of $(Y_e - t_0 Y_e)$:

$$Y_e = c_0 + MPC(1 - t_0)Y_e + I_0 + G_0$$

(2) Put all the terms with Y_e on the left hand side:

$$Y_e - MPC(1 - t_0)Y_e = c_0 + I_0 + G_0$$

(3) Factor out Y_e from the left hand side:

$$[1 - MPC(1-t_0)]Y_e = c_0 + I_0 + G_0$$

(4) Solve for Y_e:

$$Y_e = \frac{c_0 + I_0 + G_0}{1 - MPC\,(1 - t_0)} \qquad \text{(Equation 1)}$$

Equation 1 is the key "reduced form" of the system of equations that compose this model because it ***expresses the equilibrium value of the key endogenous variable as a function of exogenous variables alon***e. Equation 1 can be evaluated at any combination of exogenous variables in order to determine Y_e. We can also write reduced form expressions for the other endogenous variables, C and AD, but they are not as important.

A CONCRETE EXAMPLE:

Suppose: $C = 200 + 0.8(Y - 0.0625Y)$
$I = 200$
$G = 100$

Then, applying the steps in the recipe we have:

STEP (1): Write the structural equation(s) and equilibrium condition(s)

For this problem, we have:

Structural Equations:

$C = 200 + 0.8(Y - 0.0625Y)$
$I = 200$
$G = 100$
$AD = 200 + 0.8(Y - 0.0625Y) + 200 + 100$

and

Equilibrium Condition:

$Y = AD$

STEP (2): Force the structural equations to obey the equilibrium conditions.

We do this by writing:

$Y_e = 200 + 0.8(Y - 0.0625Y_e) + 200 + 100$

STEP (3): Solve for the equilibrium value of the endogenous variable; that is, **rearrange** the equation in Step 2 so that you have Y_e by itself on the left-hand side and only exogenous variables on the right.

Y_e can be found by solving the equilibrium condition for Y as follows:

$$Y_e = 200 + .8(Y_e - .0625Y_e) + 200 + 100$$

$$Y_e = 200 + .8(1 - .0625)Y_e + 200 + 100$$

$$Y_e - .8(1 - .0625)Y_e = 200 + 200 + 100$$

$$[1 - .8(1 - .0625)]Y_e = 200 + 200 + 100$$

$$Y_e = \{200 + 200 + 100\}/[1 - .8(1 - .0625)]$$

$$Y_e = \$2000$$

NOTE: The "reduced form equation" (general solution) for Y_e given by Equation 1 above evaluated at c_0=200, MPC=0.8, t_0=0.0625, I_0=200, and G_0=100 also yields Y_e = \$2000.

At any other level of output, AD will not *exactly* equal actual GDP. This is easy to verify. Pick any other output level—say, Y = 1500. The value of AD at that level of income is 200 + (.8)(1500 - .0625*1500) + 200 + 100 = 1625. (*Check and make sure you follow this calculation.)*

Clearly, aggregate demand exceeds output; so inventories will fall, signaling increased production in the next period. Let's say firms produce \$100 more in output so that output is now \$1600. Aggregate demand when income is \$1600 is \$1700. Notice that the extra \$100 in output resulted in an increase of only \$75 in aggregate demand? Where did the other \$25 go? \$5 went to the government in taxes and \$20 is saved (not spent on goods and services) by consumers. Notice also that we are closer to equilibrium in that the reduction in inventories was \$125 at Y=\$1500 and now, at Y=1600, it's only \$100.

Values of Y other than \$2000 will yield a situation where inventories must either increase or decrease resulting in changes in Y and, therefore, cannot be equilibrium values of Y.

Understanding the Equilibration Process

Now that we know how to find the equilibrium solution, let's take a closer look at how we get to equilibrium using the numerical example above for concreteness.

$$AD = 200 + (.8)(.9375*Y) + 300 = .75*Y + 500 \qquad \text{[Summary of the structural equations]}$$

$$Y = AD \qquad \text{[Equilibrium condition]}$$

Let's suppose that initially the value of Y (output/income) is $900. If income is $900, then consumers, government, and firms together will demand (0.75*$900)+$500 = $1175 worth of output. When that happens, firms will have to dip into their inventories to the tune of $900 - 1175 = - $275 in order to meet demand. (Remember: A negative number means unintended inventory depletion or a decline in inventory stocks below the optimal level.) In the next period, firms will tell their factories to increase production to meet the higher than anticipated level of demand as well as to provide extra stock to build up their inventory cushion.

How much more will firms produce in the next period in response to the shortfall of - $275? The answer to that determines the type of equilibrium process that we will observe. Suppose that firms decide to increase production by the **exact amount** of the inventory depletion that occurred in the last period. Then next period's output (Y_1) will be $900 + $275 = $1175. If income is $1175, then AD will be ($1175*.75) + 500 = $1381. Once again, aggregate demand exceeds output produced and firms must draw down their inventories—this time by - $206. By increasing output up to the previous level of aggregate demand, firms have raised income and therefore increased aggregate demand.

If firms increase production again by $206 (to $1381), aggregate demand will still exceed output (though not by as much). See the table below for the rest of the story.

Time period	Y	AD	ΔInventories
0	900	1175	-275
1	1175	1381	-206
2	1381	1536	-155
3	1536	1652	-116
4	1652	1739	-87
5	1739	1804	-65
6	1804	1853	-49
7	1853	1890	-37
8	1890	1917	-28
9	1917	1938	-21
10	1938	1954	-15
11	1954	1965	-12
12	1965	1974	-9
13	1974	1980	-7
14	1980	1985	-5
15	1985	1989	-4
16	1989	1992	-3
17	1992	1994	-2
18	1994	1995	-2
19	1995	1997	-1
20	1997	1997	-1
21	1997	1998	-1
22	1998	1999	-0.49
23	1999	1999	-0.37
24	1999	1999	-0.28
25	1999	1999	-0.21
n	2000	2000	0

The equilibration process assuming that firms decide to increase production by the **exact amount** of the inventory depletion that occurred in the last period.

The Phase Diagram

Looking at the table on the previous page, it is clear that the equilibration process is **convergent** and **direct**. It is *convergent* because we move toward equilibrium. It is *direct* because there is no overshooting, then undershooting. From your work with phase diagrams, you might remember that directly convergent equilibration processes have phase lines with slopes between 0 and 1.[3]

Before we draw the phase diagram for this model, let's see if we can understand what the difference equations that describe the system must look like.

We know that output this period is equal to output last period plus the change in output. So, we can write:

$$Y_{t+1} = Y_t + \Delta Y_{t+1}$$

We have assumed that "firms decide to increase production by the **exact amount** of the inventory depletion that occurred in the last period." That means that the change in output is defined as

$$\Delta Y_{t+1} = \text{rho} * (Y_t - AD_t)$$

The assumption that "firms decide to increase production by the **exact amount** of the inventory depletion that occurred in the last period" boils down to assuming that rho equals minus one. Then, given any amount of the change in inventory, $Y_t - AD_t$ (accumulation if positive, no change if zero, and decumulation if negative), the change in output is equal to minus the change in inventory. For example, if at time t output is 700 and aggregate demand is 750, then firms will increase output in period t+1 by

$$-1 \cdot (700 - 750) = 50$$

The Simple Keynesian Model has an equilibration process that is captured in a difference equation by the following:

$$Y_{t+1} = Y_t + \Delta Y_{t+1}$$
$$\text{where } \Delta Y_{t+1} = \text{rho} * (Y_t - AD_t)$$

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3 For your information, the relationship between the slope of the phase line and the equilibration process that you derived in the last lab is summarized below:

Slope of phase line	**Type of Equilibration Process**
Less than -1	Oscillating divergence
Equal to -1	Uniform oscillation
Between -1 and 0	Oscillating convergence
Equal to 0	Instantaneous convergence
Between 0 and +1	Direct convergence
Equal to +1	No movement
Greater than +1	Direct non-convergence

Notice the similarities to the Profit Equilibration Model we explored last time. As before, we are NOT saying that rho must be some value. On the contrary, rho can be any value and it is up to us to determine, in a particular economy at a particular time what that value is.

Recall that the phase line plots values of the endogenous variable at time t on the x-axis against values of the endogenous variable at time t+1 on the y-axis.

In this example, we will plot the variable Y_t on the x-axis and the variable Y_{t+1} on the y-axis. We can draw pictures of the phase diagram next to the canonical "Keynesian cross" graph:

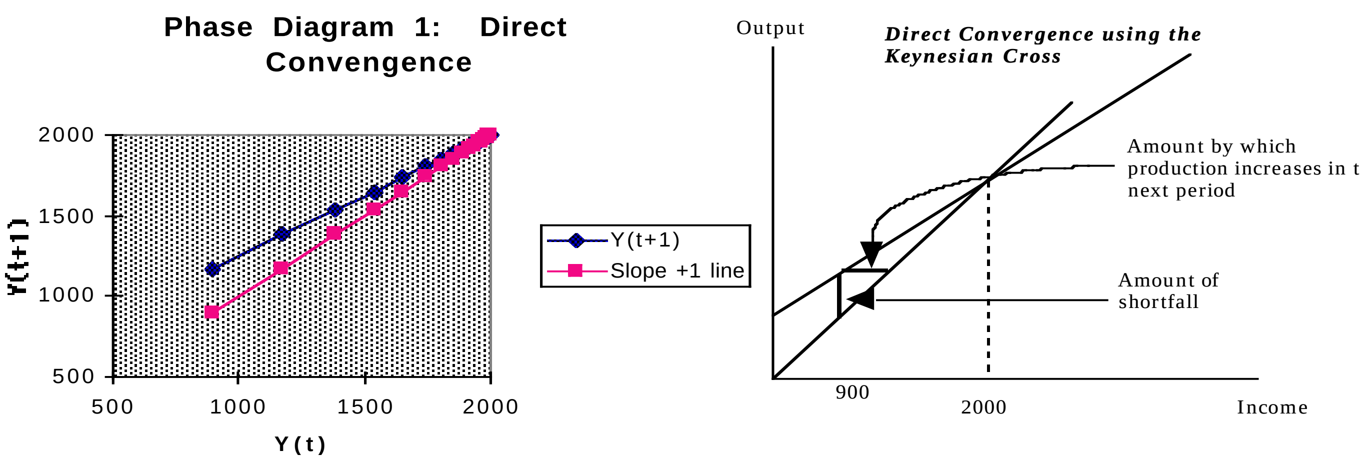

Figure 2: The Equilibration Process when rho = - 1

Notice how the lesson of the phase line remains intact:

Its slope is clearly positive, but less than + 1 (in fact, it is 0.75) and this results in

- A Stable Equilibrium
- Direct Convergence to the equilibrium solution

As for speed, the closer to 0 the slope of the phase line gets, the faster the system reaches equilibrium. The slope of the phase line answers the three questions we are interested in about the equilibration process.

Another Possibility for the Equilibration Process

Of course, rho = - 1 is not the only possibility for the equilibration process. Suppose that firms were much more responsive to the larger than anticipated demand and produced a lot more than simply the decumulation in inventories, say, six times as much.

As before, the difference equation would be:

$$Y_{t+1} = Y_t + \Delta Y_{t+1}$$
$$\text{where } \Delta Y_{t+1} = \text{rho} * (Y_t - AD_t)$$

Now, however, rho = - 6!

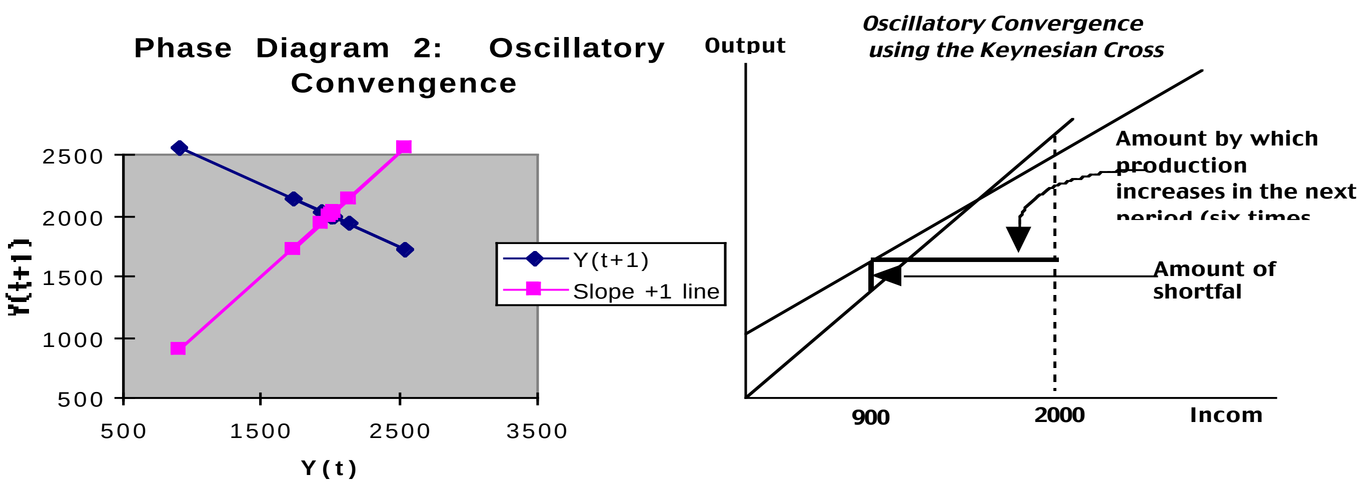

Figure 3: The Equilibration Process when rho = - 6

Notice how the lesson of the phase line remains intact:

Its slope is clearly less negative, but less than - 1 (in absolute value) (in fact, it is - 0.45) and this results in

- A Stable Equilibrium
- Oscillatory Convergence to the equilibrium solution

As for speed, the closer to 0 the slope of the phase line gets, the faster the system reaches equilibrium. The slope of the phase line answers the three questions we are interested in about the equilibration process.

III) Comparative Statics

Now that we have found the equilibrium level of output (and analyzed Y_e verbally, graphically and mathematically) and discussed the equilibration process, we will focus on an examination of how changes in exogenous variables affect Y_e. In particular, we will consider the following two types of problems:

1) If an exogenous variable (c_0, MPC, t_0, I_0, G_0) changes, how will Y_e (the endogenous variable) change?

2) How and by how much do we need to change G_0 and/or t_0 (our policy variables) to get a desired change in Y_e?

In their basic form, these are the two fundamental questions asked of macroeconomic models. There are always variables that are determined by forces outside the model and, hence, are considered given and unresponsive to changes in other variables in the model. In contrast to these exogenous variables, endogenous variables are determined by the interplay of forces within the model. A "shock" is a change in one exogenous variable (holding all others constant). The natural questions are: (1) How does a shock affect an endogenous variable? and, (2) What kind of a shock would be needed to get a given effect on an endogenous variable?

Note that the questions can be answered qualitatively or quantitatively. The former implies an answer based on the *direction* of the change (up or down, higher or lower, increase or decrease); while the latter requires a more precise measure, for both *direction and magnitude* are needed (*how much* higher or lower). Economists typically give both qualitative and quantitative answers to questions, depending on the purpose at hand.

ASIDE:

Before we begin consideration of these two questions, an oft-leveled criticism should be mentioned. "Comparative statics" analysis means that the investigator "compares" two alternative positions, the initial and new levels of the endogenous variable(s). The macroeconomist notes, for example, the initial equilibrium value of output, determines the shock (say, an increase in government spending), calculates the new equilibrium value of output, and compares the two. Economists focus on initial and new (or final) in order to facilitate comparison. A pure comparative statics methodology implies no consideration of the process by which the economy moves from initial to new equilibria.

Economists are paying increasing attention to issues such as process, speed of response, and the microeconomic details of how the economy moves from initial to new equilibria. We explored the equilibration process of this model in some detail above and we will continue our investigation into how equilibrium is reached in the Simple Keynesian Model in the next chapter.

Question 1: What is the effect of a shock on Y_e?

In the model above, assume that G, for some reason, increases by \$100, *ceteris paribus*. What is the effect of this shock on Y_e?

Macroeconomists have several ways of solving this problem. We could recalculate Equation 1—changing G from its old \$100 value to its new \$200 value. Calculating, the new equilibrium level of output, Y_e', is \$2400 (see Figure 4 below).

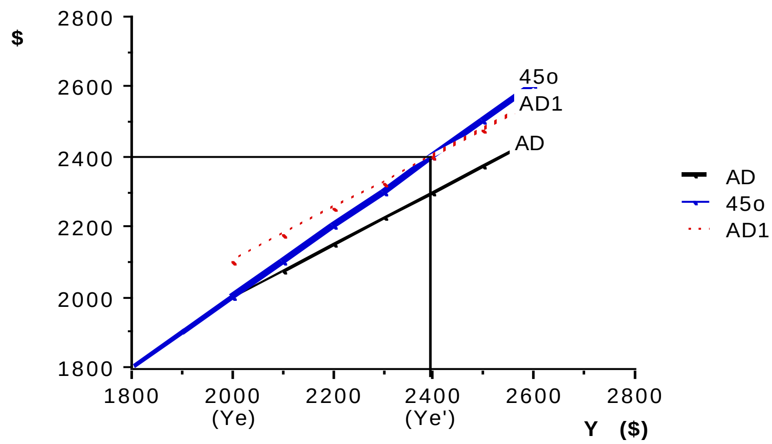

FIGURE 4: The Effect on Y_e of an Increase in G_0

The fact that the equilibrium level of income increased by more—four times more—than the increase in government spending is evidence of the "multiplier." In this case, the Government Spending Multiplier is 4—every \$1 increase in G results in a \$4 increase in Y_e.

A second method of calculating the new Y_e explicitly derives the multiplier as a derivative of the reduced form and then uses it to determine the new equilibrium level of income. Equation 1,

$$Y_e = \frac{c_0 + I_0 + G_0}{1 - MPC\,(1 - t_0)},$$

tells us the equilibrium level of income given the exogenous variables on the left hand side. We want to determine how Y_e will change for a given change in G. Mathematically, we want to find the **derivative** of Y_e with respect to G; that is, the rate of change in Y_e with respect to G:

$$\frac{dY_e}{dG} = \frac{1}{1 - MPC\,(1 - t_0)}$$

$$\left.\frac{dY_e}{dG}\right|_{\substack{MPC=.8 \\ t_0=6.25\%}} = \frac{1}{1 - .8 \cdot (1 - .0625)}$$

Here we have a general solution to Type 1 problems concerning G (i.e., what is the effect on Y_e of a given change in G?). We need not recalculate for every change in G—we simply take the given change in autonomous expenditure, multiply by 4, and add it to the old equilibrium level of output.[4]

What we are actually doing is examining how the endogenous variable, Y_e, responds to a change in an exogenous variable, G. Therefore, we should be able to show this relationship as a presentation graph:

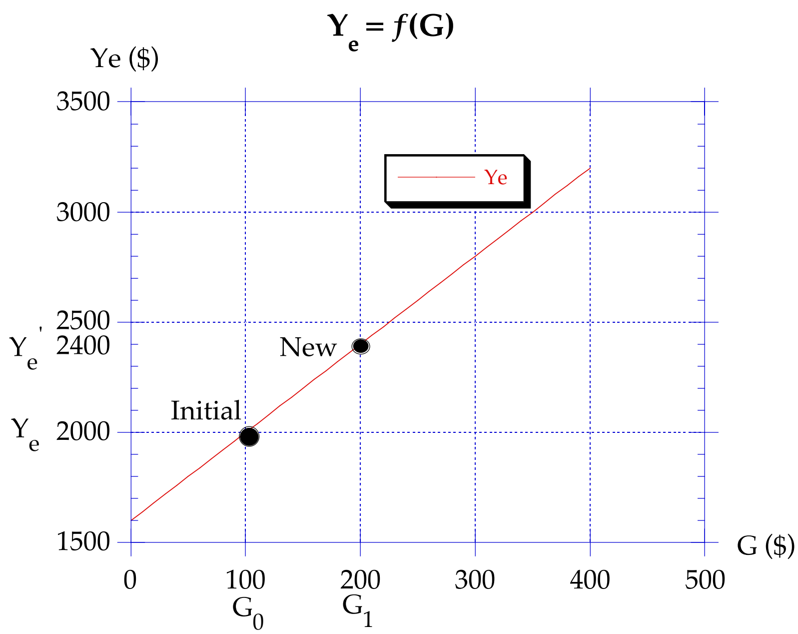

Figure 5: Equilibrium Output as a Function of Government Spending

You should see the similarity between the presentation of this relationship and that of the endo*=f(exo) graphs in optimization problems. Note the fact that this graph can be used to easily answer "Given a shock in G, what's the new Y_e?" questions. The graph is read vertically: given G, the line immediately reveals Y_e. This is exactly the same as our procedure for reading presentation graphs in optimization problems.

Of course, there are some crucial differences too. We are not tracking optimal solutions, but equilibrium positions. To be off the line in the graph above is to be on a value that has a tendency to change and that will be drawn to the line (like iron filings to a magnet). Thus, presentation graphs in equilibrium models offer a different analogy than those in optimization problems. Instead of a "ridge line across connecting hilltops," a line or curve in an equilibrium presentation graph is better seen as a MAGNET that attracts non-equilibrium values (if the system is stable).

4 For the concrete example at hand, ΔG=+\$100, multiplier = 4, and initial Y_e = \$2000, we know that ΔY_e = \$400 (= 4 * \$100) and new Y_e = \$2400 (= initial $Y_e + \Delta Y_e$ = \$2000 + \$400).

Finally, note also the linear relationship between Y_e and G. This has important implications for computing comparative statics results via calculus versus directly calculating equilibrium values for finite shocks. Because the relationship is linear, both the Method of Reduced Form (via calculus) and the Method of Actual Comparison (via, say, the Comparative Statics Wizard) will yield exactly identical results.

You should derive dY_e/dI and dY_e/dc (Note: this is lower case c—the intercept term in the consumption function) and confirm that they are identical to the multiplier above. Intuitively, these multipliers are identical because G, I and c affect AD the same way—i.e., a shift of the intercept and no change in the slope. For this reason, graphs of the relationship between Y_e and I or c will be linear.

On the other hand, changes in the tax rate or MPC change the slope without changing the intercept. Thus, we would expect a different multiplier. Computing dY_e/dt is conceptually identical to the dY_e/dG work above, but the actual calculation of the derivative is more difficult because t appears in the denominator.

$Y_e = f(t)$ and finding dY_e/dt

Remembering that $y = u/v = uv^{-1}$ and the chain rule (if $y=f(v)$ and $v=g(x)$ then $dy/dx=(dy/dv)(dv/dx)$), we can mitigate the pain of solving for the tax multiplier.

First, we rewrite the reduced form,

$$Y_e = \frac{c_0 + I_0 + G_0}{1 - MPC(1 - t_0)} = (c_0 + I_0 + G_0)(1 - MPC(1 - t_0))^{-1}$$

Then, we take the derivative with respect to t in order to find the tax multiplier,

$$\frac{dY_e}{dt} = -\frac{MPC(c_0 + I_0 + G_0)}{\left[1 - MPC(1 - t_0)\right]^2}.$$

Evaluating this equation at the initial values of the exogenous variables gives the following numerical solution:

$$\left.\frac{dY_e}{dt}\right|_{\substack{MPC=.8\\ c_0=200\\ I_0=200\\ G_0=100\\ t_0=.0625}} = -6400$$

This tells us that an infinitesimal increase in t will lead to a 6400-fold decrease in Y_e. The minus sign matters—it says that INcreases in t lead to DEcreases in Y_e. We say "infinitesimal increase" instead of a one percentage point increase in t (from, say, 6.25% to 7.25%) because Y_e is non-linear in t. This means that the tax multiplier depends on both the magnitude of the increase and the initial value of the tax rate. It is not true that, as in the case of a change in G, that the tax multiplier will always be the same.

The non-linearity of Y_e as a function of t is captured in the presentation graph of the relationship:

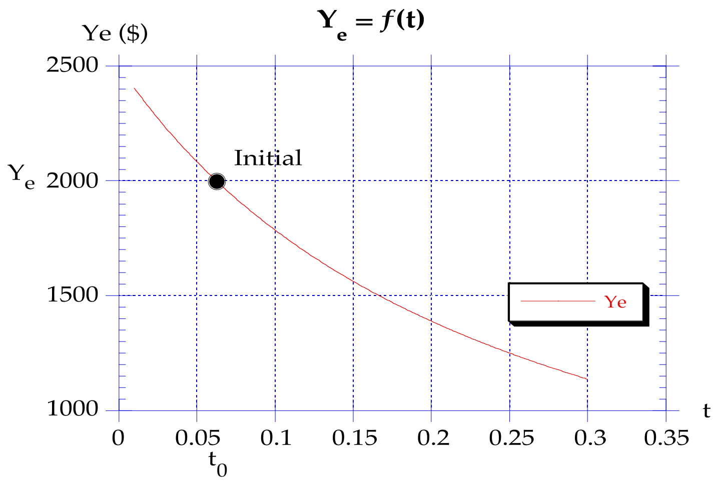

Figure 6: Equilibrium Output as a Function of the Tax Rate

Question 2: What is the shock needed to generate desired Y_e?

The second major question we ask in macroeconomics concerns the choice of policy tools to move Y_e to a desired level. In Keynesian models, a "full-employment" level of output, Y_f, is often postulated and the policy maker is responsible for ensuring that the economy's equilibrium level of output matches the given full-employment level.

For example, considering our first concrete model above:

$$C = 200 + .8(Y - .0625Y)$$
$$I = 200$$
$$G = 100$$

we found that Y_e = \$2000. Suppose, however, that Y_f = \$2500.

The policy maker needs to reduce the GDP Gap ($Y_f - Y_e$) to zero. Usually, c and MPC are considered beyond the reach of systematic government manipulation. The government could launch a campaign to encourage consumer spending—thereby shifting and/or rotating the consumption function; but the analysis usually focuses on G, t and I. In the Simple Keynesian Model, moreover, there is no way to influence I—leaving us with only the fiscal policy tools of G and t.

The policy maker must calculate the multiplier in order to determine the correct "shock" that needs to be administered.[5] We found above that

$dY_e/dG = 1/[1-MPC(1-t)]=4$

Above, we knew the change in G and by multiplying by 4, found the change in Y_e. In this case, however, we know the desired change in Y_e, that is,

$\Delta Y_e = Y_f - Y_e = \$500$

and seek the needed change in G. In other words, we have:

$\$500/dG = 4$
$dG = \$125$

A \$125 increase in G will increase Y_e by \$500, which will move the equilibrium level of output to \$2500. Since this level of output is the full-employment level, the policy maker has accomplished her task. Figure 7 provides a graphical representation of this example.

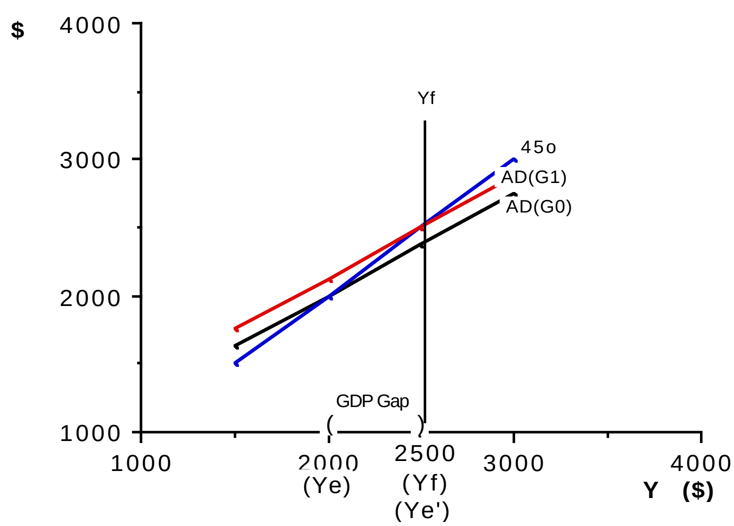

FIGURE 7: Reaching Y_f by Increasing G

5 A process of iteration using the "recalculate for every given shock" method is possible, but clearly tiring! Unless, of course, Excel's Solver and the Comparative Statics Wizard are available . . .

Note that the government is now running a deficit. Tax revenues have increased (because national income increased) to (.0625)($2500) = $156.25; while expenditures have risen to $225 (i.e., the previous level of G plus the additional $125 in expenditures). This $68.75 budget deficit is not problematical in this model because there are no mechanisms by which the deficit can affect Y_e.

The policy maker could also ***lower the tax rate*** in an attempt to increase AD and, consequently, Y_e. In this example, the policy maker would have to set t=0 in order to get the required change in Y_e. The reader is left with the task of working out the mathematics of the solution, but a graphical representation (Figure 8) is shown below. (Hint: You cannot use the multiplier because it is non-linear; instead, substitute the desired level of income Y_f into the reduced form and solve for the tax rate t.)

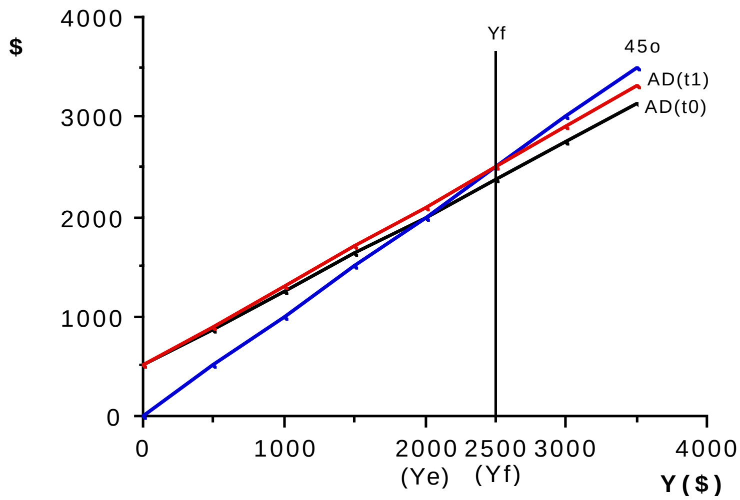

FIGURE 8: Reaching Y_f by Decreasing t

For the sake of completeness, we should mention that a combination policy could also be used. There are many mixes of changes in G and changes in t that will force Y_e=Y_f. In fact, it is even possible to find a G/t mix that simultaneously balances the budget and eliminates the GDP Gap. In this example, increasing G by $400 to G = $500 and increasing taxes to t=20% yields $Y_e = Y_f$ = $2500 and a balanced budget (revenues = $500 = expenditures). You should be able to show graphically the separate effects of the changes in G and t and the total, or combined, effect of these changes.

Summary and Conclusion:

We have constructed and analyzed the Simple Keynesian Model. You should understand how to find the initial solution, how equilibrium is determined and how changes in exogenous variables affect the equilibrium level of output.

To Review:

Equilibrium: Any given level of income will determine a level of aggregate demand. If that level of aggregate demand does not exactly exhaust actual GDP, there is a tendency for output (and, by definition, income) to change:

If AD>actual GDP ==> depletion of inventories ==> increased Y next period

If AD<actual GDP ==> addition to inventories ==> decreased Y next period

If AD=actual GDP ==> unchanged inventories ==> unchanged Y next period

The last case is equilibrium because there is no tendency to change—the economy will continue producing the same level of output in every time period.

Equilibration Process: the nature of the equilibration process depends on how responsive firms are to accumulations /decumulations in inventories. You learned how different assumptions about the adjustment process generated different types of equilibration and that the phase line *consistently* and immediately reveals if the equilibrium is stable, its type, and its speed.

Comparative Statics: Changes in any exogenous variable (i.e., c, MPC, t, I, G) will change the key endogenous variable (Y_e). Graphically, such changes are represented by shifts in the AD function (in the case of c, I and G) or rotations along the intercept (in the case of MPC and t).

Increases in c, I, G and MPC lead to increases in Y_e; however, increases in t lead to decreases in Y_e. Mathematically, we are able to derive not only the qualitative, but also the quantitative change in Y_e for a given change in an exogenous variable. In order to do this, we can recalculate Y_e by including the new values of the exogenous variable; or, if the relationship between Y_e and the exogenous variable is linear, we can, having found dY_e/dExogenousVariable, apply the multiplier to a given change.

The two fundamental questions: In macroeconomic models, we ask the following two fundamental questions:

(1) How do changes in exogenous variables (in this case, we examined G and t) affect the endogenous variable(s) (in this case, Y_e)?

(2) What can policy makers do (in this case, G and t are the policy variables) to eliminate the GDP Gap?

What Lies Ahead?

In the next chapter, we will continue our analysis of the Equilibration Process—phase diagrams; types and speed of equilibrium. In addition, we offer further exploration of comparative statics with Excel's Solver and the Comparative Statics Wizard, including a return to the often confusing distinction between linear and non-linear reduced forms (coupled, of course, with a discussion of "d" versus "Δ").