Chapter 10: Constrained Optimization *via* Calculus

Introduction

You have learned how to solve one-variable and two-variable unconstrained optimization problems. We now proceed to the next level: solving two-variable problems in which there is a **constraint** on the actions of the optimizing agent.

Another way of saying this is that there is a restriction on the values of the endogenous variables that may be chosen. We will introduce you to a nifty technique that transforms the constrained problem into an unconstrained one. Once you've done this, you can apply the familiar five-step process for solving an unconstrained problem.

Our goal here is to teach you a **recipe** for solving problems, not to teach you the mathematics behind it—that job we will leave to your mathematics professors. In economics, if you follow the recipe, you will get the right answer.

The Example Problem

On a given evening, Wally enjoys the consumption of cigars (C) and brandy (B) according to the utility (U) function

$$U = 20C - C^2 + 18B - 3B^2$$

It is possible for Wally to consume only part of a cigar (he smokes 1/2 or maybe .743267. . . of a cigar) or brandy (by drinking 1/3 or possibly .23784. . . of an ounce).

Our Tasks Today:

We will first suppose that there is no constraint on the amount of cigars and brandy that Wally can choose: he can pick any amount of C and any amount of B to maximize his utility function.

How much of each will he consume? What's the max U? We can apply the familiar five-step solution strategy for a two-variable unconstrained problem to find out.

We will then impose a constraint on Wally's choices. We will see how the constraint is incorporated into the problem and how the solution differs from the unconstrained problem. We will revisit the linear/non-linear reduced form issue and learn an important lesson about the Method of Actual Comparison versus the Method of the Reduced Form.

Solving the Unconstrained Problem with Calculus

Let's suppose that we have set up the problem and understand the goal (maximize utility), the endogenous variables (B and C) and the exogenous variables (the numbers ("20", "2," etc. in the utility function).

We are ready to FIND THE OPTIMAL SOLUTION.

Step 1: Write out the problem mathematically

$$\max_{C,\,B} U = 20 \cdot C - C^2 + 18 \cdot B - 3 \cdot B^2$$

Step 2: Take the partial derivatives of the objective function.

$$\frac{\partial U}{\partial B} = 18 - 6B$$

$$\frac{\partial U}{\partial C} = 20 - 2C$$

Step 3: Set the partial derivatives equal to zero and put a * superscript on the endogenous variables (B and C) to indicate that they now represent the optimal values of the endogenous variables:

$$\frac{\partial U}{\partial C} = 20 - 2C^* = 0$$

$$\frac{\partial U}{\partial B} = 18 - 6B^* = 0$$

Step 4: Solve for the optimal values of the endogenous variables:

$$C^* = 10 \text{ cigars}$$
$$B^* = 3 \text{ ounces of brandy}$$

Notice how easy this step is since the 2 first-order conditions (in Step 3) are UNRELATED.

Usually, the first-order conditions will be RELATED and a series of algebra moves will be necessary to solve for the endogenous variables.

Step 5: Plug the optimal values of the endogenous variables into the objective function in order to calculate the maximum utility:

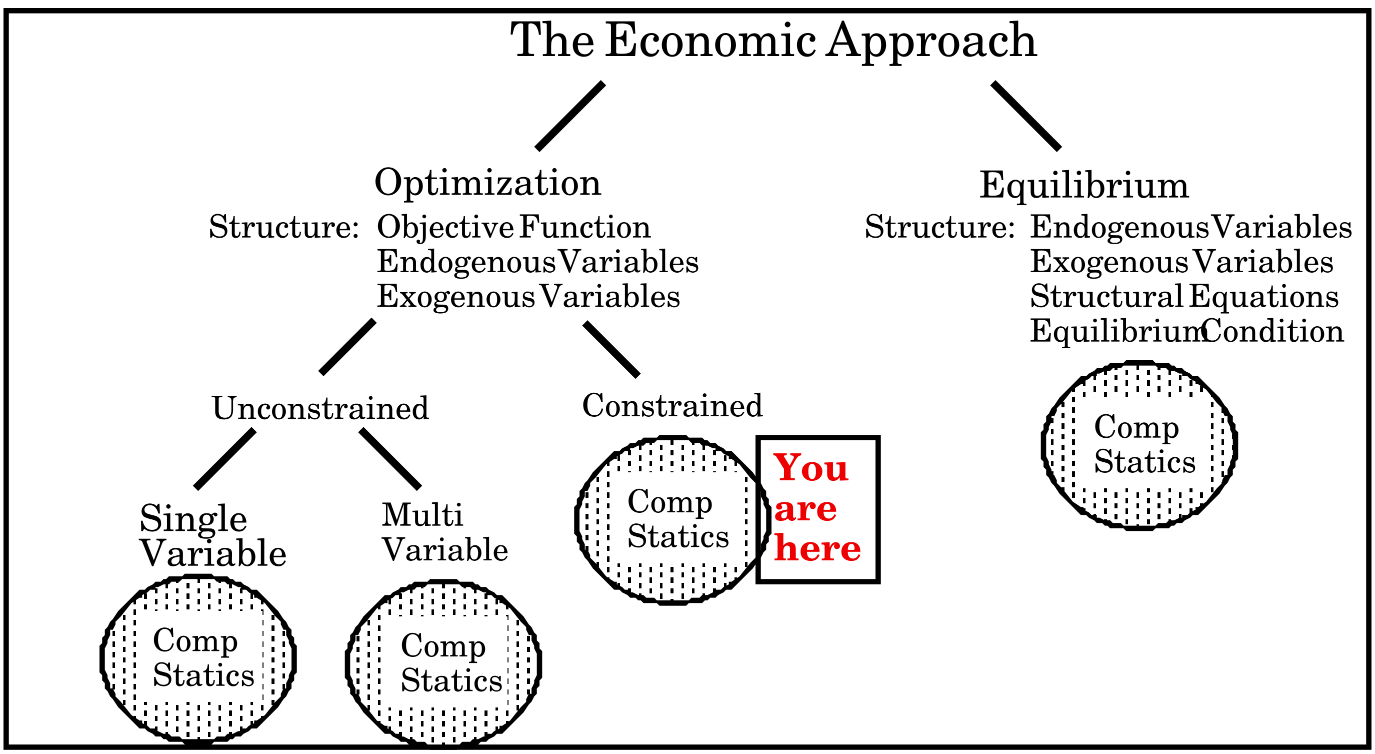
$$U^* = U|_{B=B^* \text{ and } C=C^*} = 20[10] - [10]^2 + 18[3] - 3[3]^2 = 127$$

This means that Wally enjoys a maximum utility level of 127 utils when he can consume as many cigars and brandy as he wishes. He cannot do any better than this.

What's Next?

From here, we would normally proceed to explore the COMPARATIVE STATICS properties of the solutions; that is, we would investigate how C*, and B* and U* change for given shocks. Of course, we could use the Method of Actual Comparison either via cumbersome calculation with pencil-and-paper or, much more easily, with the aid of Excel's Solver. Alternatively, we could solve a more general version of the problem (with letters instead of numerical values for the exogenous variables) and then apply the Method of the Reduced Form via Calculus. But all of that would merely repeat what we already know.

Today, however, we will turn to a constrained version of the problem and learn a new technique for solving it. This will serve as our introduction to CONSTRAINED OPTIMIZATION PROBLEMS.
To be sure you understand where we are headed, take a look at the familiar diagram below—the next few chapters are located firmly in the CONSTRAINED optimization section of the Economic Approach:



The Constrained Version of the Cigar/Brandy Utility Maximization Problem

After going to the doctor for a persistent cough, Wally is given strict orders to limit his total consumption of cigars and brandies, C+B, to 5. In other words, the doctor has put a **constraint** on Wally's actions.

Obviously, Wally's unconstrained optimal solution violates the doctor's orders: $B^* + C^* = 13$, which is a greater than the upper limit of 5. Given that he wants to maximize his utility function, but also wants to obey the doctor's orders, how many cigars and brandies will Wally consume this evening? Put in other words, *what is the best Wally can do given the constraint?*

The Solution Method in Words. The trick we will show you to solve this problem involves using the constraint ($\mathbf{5 = C + B}$) to make an *additional* marginal optimization condition. Once you have done this, you solve the resulting system as if it were a regular multi-variable unconstrained optimization problem. This nifty trick was invented in the eighteenth century by the French mathematician, Joseph Louis Lagrange (1736-1813) (sometimes spelled "LaGrange"). The **Lagrangean Method** for solving constrained optimization problems is named after him.

Basically, the Lagrangean method requires the modification of the first step in solving an unconstrained problem, writing out the objective function. The modification is this:

- **Write the equation for the constraint so that ZERO appears on the righthand side (so that EXOGENOUS LIMIT - FUNCTION OF ENDOGENOUS VARIABLES = 0).**
- **Multiply the lefthand side of the constraint equation (the non-zero part) times a NEW VARIABLE, λ (an as yet undetermined number called *the Lagrangean multiplier*).**
- **Add the resulting term to the objective function.**

When you proceed to Step 2, taking the derivatives with respect to the endogenous variables, you treat the new variable λ just like any of the endogenous variables. When you solve for the optimal values of the endogenous variables, you also solve for the optimal value of λ, λ^*. We will defer the economic interpretation of λ^* until later; for the moment, just think of it as a means toward an end—solving constrained optimization problems.

Let's work through an example.

The Lagrangean Method Applied to the Problem

Step 1: Write the problem mathematically as an unconstrained problem by utilizing the Lagrangean

(a) Write out the constraint in the form Constraint = 0.

The constraint is $5 = C + B$,[1] which we rewrite $5 - C - B = 0$. We could write it $C + B - 5 = 0$—it works that way too, but to be consistent, we'll always write it Exogenous Limit - Function of Endogenous Variables.

(b) Multiply the lefthand side of the constraint by λ, to get

$$\lambda \bullet (5 - C - B)$$

> Note that if we can somehow be assured that the constraint will hold, i.e., that C and B equal 5, then this term is equal to zero regardless of the value of λ.

(c) Add the new term to the objective function to build a new, composite function:

$$20C - C^2 + 18B - 3B^2 + \lambda(5 - C - B)$$

Strictly speaking, this expression is not utility, U, since we have added another term. We have transformed the utility function into something called a Lagrangean function, usually denoted in mathematics textbooks as a fancy-script, cursive L, $\mathcal{L}$. We will treat the Lagrangean function just like the objective function in the unconstrained problem.

$$\max_{B,C,\lambda} \mathbf{L} = \mathbf{20C - C^2 + 18B - 3B^2 + \lambda(5 - C - B)}$$

Step 2: Take the partial derivatives with respect to the THREE endogenous variables, B, C, and λ.

$$\frac{\partial L}{\partial C} = 20 - 2C - \lambda$$

$$\frac{\partial L}{\partial B} = 18 - 6B - \lambda$$

$$\frac{\partial L}{\partial \lambda} = 5 - B - C$$

- Notice that you now have THREE first-order conditions instead of the two for the unconstrained problem.
- Notice also that the derivative with respect to lambda simply repeats the constraint. This is always the case.

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[1]Actually, the constraint is $C + B \leq 5$ rather than strict equality; but since Wally likes both cigars and brandy at these levels of consumption, we know that he will not be maximizing utility unless he consumes as much as he can. Hence, we know that the constraint will hold with strict equality.

Step 3: Set the partial derivatives equal to zero and put stars (*) next to the endogenous variables to identify them as the optimal values.

$$20 - 2C^* - \lambda^* = 0$$
$$18 - 6B^* - \lambda^* = 0$$
$$5 - B^* - C^* = 0$$

This gives you THREE equations in THREE unknowns.

Step 4: Solve for the optimal values of the endogenous variables.

From the first two equations, you get $20 - 2C^* = 18 - 6B^*$. Solving for C^*, you get $1 + 3B^* = C^*$. Substituting for the C^* in the third equation, you obtain $B^* + 1 + 3B^* - 5 = 0$. Solving for B^*, you get $B^* = 1$. This means that $C^* = 4$. Substitute either B^* or C^* into one of the first two equations and you find that $\lambda^* = 12$:

$$B^* = 1$$
$$C^* = 4$$
$$\lambda^* = 12$$

The equations were RELATED so it took a little algebra to solve for optimal values of the endogenous variables.

Step 5: Evaluate the objective function (NOT the Lagrangean) at the optimal value of the endogenous variables.

Objective function* = f(endogenous variables* | exogenous variables)

The objective function in our case is still **utility**, not the made-up, composite Lagrangean function—that's just a means to solving the problem. You plug in the optimal values of B^* and C^* into the original utility function to find out Wally's optimal level of utility given the constraint.

$$U^* = 20C^* - C^{*2} + 18B^* - 3B^{*2} = 20(4) - (4)^2 + 18(1) - 3(1)^2 = 79$$

Compared to the previous level of utility for the unconstrained problem (which yielded a maximum utility of $U^* = 127$), we see that following the doctor's orders has cost Wally 127 - 79 = 48 utils this evening (of course, he is less likely to get lung cancer or cirrhosis of the liver when he is 56!).

ASIDE: Notice how the maximum value function of the Lagrangean is also 79 since the $\lambda^*(5 - C^* - B^*)$ term equals zero. It is more logically correct, however, to find U^* and to think of the Lagrangean as a MEANS that is discarded once the solution is found. The optimal value of the Lagrangean Multiplier, λ^*, however, is not discarded for it will prove to have a powerful use.

BEING CAREFUL WITH THE UNITS:

We will break, for a moment, with the time honored tradition in economics of being incredibly sloppy with units of variables. We will painstakingly detail the units of each variable we are considering in the cigar/brandy problem. The pay-off will be a much better understanding of λ^*.

First, the objective function is measured in "utils." So, if C=1 and B=1, we would have $20[1] - [1]^2 + 18[1] - 3[1]^2 = 34$ utils. As we said before, C is number of cigars and B is ounces of brandy. That means the coefficient "20" is an exogenous variable that is measured in utils/number of cigars—so that when you multiplied it by C in number of cigars, you'd get utils. In addition, there must be a coefficient "- 1" in front of the C^2 (as in, $20C - 1C^2$) that is an exogenous variable measured in utils/number of cigars^2—so that when you multiplied it by C^2, which is in number of cigars^2, you'd get utils. Similarly, the "18" and "-3" coefficients must be measured in utils per ounce of brandy and utils per ounce of brandy^2, respectively.

Turning to the constraint, we have to point out that the exogenous "total cigar and brandy limit," isn't measured in either number of cigars or ounces of brandy, but something like "units of sin." Both C and B in the equation $5 - C - B = 0$ have "1s" in front of them that are exogenous variables measured in number of cigars/unit of sin and ounces of brandy/unit of sin. These coefficients are needed to convert C and B into some common units (which we call "sin") that can be added together.

All of this is a prelude to the most important measurement of all, that of λ. If the objective function comes out in utils, then $\lambda(5 - B - C)$ must also end up in utils. We know from above that $5 - C - B$ is in units of sin. We can correctly deduce, then, that λ must be a ***rate*** measured in utils per units of sin—so that when $5 - C - B$ in units of sin is multiplied by λ in utils/unit of sin, we get utils!

You might be thinking, "Now I know why economists never sweat the units—what a mess!" Maybe, but there is an important lesson here—in every constrained optimization problem that uses the Lagrangean Multiplier Method, *λ must be a rate measured in units of the objective function per unit of the exogenous limit in the constraint.*

In the problem above, we found that $\lambda^*=12$. The question is "12 what???" The answer is, 12 utils/unit of sin. Keep that in mind when we discuss the interpretation of λ^* below.

And remember the lesson:

LESSON: λ must be a rate measured in units of the objective function per unit of the exogenous limit in the constraint.

Comparative Statics via Calculus—The Method of the Reduced Form

Suppose that we were interested in how Wally's choices and his optimal utility level would change if the doctor were to allow him more or less latitude in exercising his bad habits. In other words, we are interested in how C*, B*, and perhaps U* change when the doctor allows him more or less cigars and brandy.

This is an application of comparative statics. We can apply the Method of the Reduced Form here to answer the question, but we need to make the problem a little more general and re-solve it in order to do it. Instead of the "concrete" limit of 5, we substitute the exogenous variable CBL (for **c**igar and **b**randy **l**imit). When we solve again for the optimal values of the endogenous variables, that new variable CBL will appear on the righthand side, allowing us to track how the optimal values of the endogenous variables change when the limit goes up or down.

We could make the problem even more general (with letters for the other exogenous variables), but since we are interested in how the Cigar and Brandy Limit affects the optimal values, we will concentrate on CBL alone. We would, of course, get the same answer by finding the general reduced form and then plugging in the same specific parameter values that we are using here.

Now we repeat the steps and re-solve the problem.

Step 1: Write the problem mathematically as an unconstrained problem by utilizing the Lagrangean

$$\max_{B,C,\lambda} \boldsymbol{L} = 20C - C^2 + 18B - 3B^2 + \lambda^*(\,\mathbf{CBL} - C - B)$$

Step 2: Take the partial derivatives with respect to the three endogenous variables.

$$\frac{\partial L}{\partial C} = 20 - 2C - \lambda$$

$$\frac{\partial L}{\partial B} = 18 - 6B - \lambda$$

$$\frac{\partial L}{\partial \lambda} = CBL - B - C$$

Step 3: Set the partial derivatives equal to zero.

$$20 - 2C^* - \lambda^* = 0$$
$$18 - 6B^* - \lambda^* = 0$$
$$CBL - B^* - C^* = 0$$

Step 4: Solve for the optimal values of the endogenous variables.

Instead of getting a number, we will get an expression which involves both numbers and CBL, the other exogenous variable that is no longer a number.

From the first two equations, we know $1 + 3B^* = C^*$, as before. Substituting into the third equation for C^*, we obtain $CBL - B^* - 1 - 3B^* = 0$, or $B^* = 0.25CBL - 0.25$. Plugging back into the expression for C^*, we get $C^* = 1 + 3(0.25CBL - 0.25) = 0.25 + 0.75\,CBL$. Finally, plugging the values for C^* or B^* into one of the first two equations gives you $\lambda^* = 19.5 - 1.5CBL$.

$$C^* = 0.25 + 0.75CBL$$
$$B^* = -\,0.25 + 0.25CBL$$
$$\lambda^* = 19.5 - 1.5CBL$$

These are reduced forms.

Note that if you plug in CBL = 5, you recover the previous "concrete" solution.

Step 5: Evaluate the objective function at the optimal values of the endogenous variables.

If we plug in the Step 4 optimal values into the expression for utility, $U^* = 20C^* - C^{*2} + 18B^* - 3B^{*2}$, we get (after a little algebra)

$$U^* = 0.25 + 19.5CBL - 0.75CBL^2$$

This is a maximum value function (a reduced form for the objective function).

Note that if CBL = 5, then $U^* = 79$—just like before!

PEDAGOGICAL HINT: There is much repetition here—the five steps for finding an optimal solution to an optimization problem via calculus are the same whether the problem is unconstrained or constrained. Once the problem is set up and Step 1 is taken care of, the mechanics of calculus are identical. Use this well-established PATTERN to create a "comfort zone" for your mind. No matter the optimization problem, you should expect to see the same things (first-order conditions, similar algebra tricks, etc.) popping up repeatedly.

Now that we have reduced forms in terms of CBL (the exogenous variable we were interested in analyzing), we are ready to perform our comparative statics analysis via the Method of the Reduced Form.

How do C*and B* change when CBL changes?

Although we could calculate and re-calculate C* and B* for different values of CBL (this is the Method of Actual Comparison) and create a nice table that showed how C* and B* varied with CBL, we can use the Reduced Forms from the previous page to quickly reveal the change in C* and B* given an infinitesimal change in CBL. This way of doing comparative statics is called the Method of the Reduced Form.

We simply take the derivative of $C^*=0.25 + 0.75CBL$ with respect to CBL and find

$$\frac{dC^*}{dCBL} = .75$$

This tells us that C* is *linear* and *increasing* in CBL.

Since it is imperative that you learn how to interpret derivatives of reduced forms, we offer you the following opportunity to understand what is going on here:

Q: Finish the statement below:

The derivative above shows that C* is linear and increasing in CBL because:

When you are done, see the last page for an answer to this question.

When the doctor allows Wally **one more** combination of cigars and brandies, he chooses to consume an additional three-fourths of a cigar.

Of course, you know that he will consume one-quarter ounce more of brandy from

$$B^* = -\ 0.25 + 0.25CBL$$

$$\frac{dB^*}{dCBL} = .25$$

This tells us that B* is *linear* and *increasing* in CBL. Similarly, if the doctor allowed Wally one LESS unit in total, he would cut back by 3/4 of a cigar and 1/4 of an ounce of brandy.

How does U* change when CBL changes?

To find this out, we take the derivative of $U^* = 0.25 + 19.5CBL - 0.75CBL^2$ with respect to CBL:

$$\frac{dU^*}{dCBL} = 19.5 - 1.5CBL$$

This derivative of the maximum value function tells you **how much maximum utility goes up when the doctor's limit goes up**.

Note that U* is *increasing* in CBL but at a *decreasing* rate. U* is non-linear in CBL.

When CBL is 5, then $dU^*/dCBL = 19.5 - 1.5(5) = 12$. This means that if the doctor allowed Wally a **little bit (as in an infinitesimally small bit)** more than 5 total cigars and brandies, his utility would go up by 12 times the increase in the limit, CBL.

"What if the doctor allows Wally a one-unit increase from 5 to 6 total cigars and brandies—would U* increase 12-fold to 91 (from 79)?"

Wow! What a great question! The answer is, "NO because U* is non-linear in CBL." There are TWO crucial points here:

- If the reduced form is LINEAR, then the Method of Actual Comparison and the Method of the Reduced Form yield equivalent results. It doesn't matter if the change in the exogenous variable is infinitesimally small (as used by the Method of the Reduced Form) or arbitrarily large (as used by the Method of Actual Comparison) since the equation is a STRAIGHT LINE. The slope is the same whether you figure the rise for a run of infinitesimally small size, .01, 1, or any size whatsoever.

- If the reduced form is NON-LINEAR, the two methods give different answers since the equation is NOT A STRAIGHT LINE. The slope will be different depending on the size of the change considered in the exogenous variable.

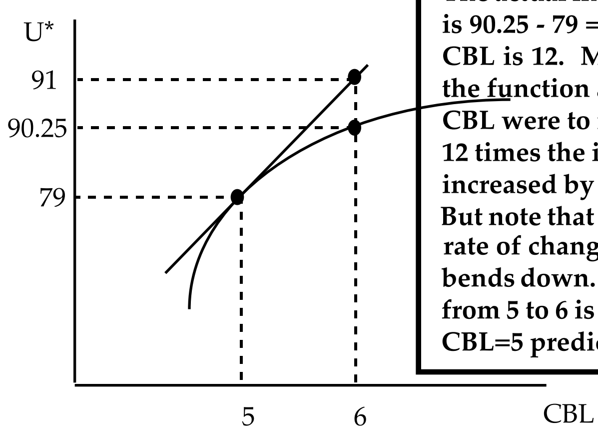
"So, if the reduced form is non-linear, which method—actual comparison or reduced form—is better?"

THE RIGHT METHOD TO USE DEPENDS ON THE QUESTION THAT IS ASKED.

Comparative statics via the Method of the Reduced Form is good for infinitesimally small changes in the exogenous variables; while the Method of Actual Comparison is better for arbitrarily large changes.

> The Method of the Reduced Form is better when you want to get a quick, fast answer to how the optimal value will change if there's a small shock.
>
> The Method of Actual Comparison is better when you are exploring the effects of a given sized shock. If your boss wants to know how profits will respond to change in price from \$6/unit to \$7/unit, use the Method of Actual Comparison.

Let's how see how this discussion applies to the question of which method is more appropriate when considering a change in CBL from 5 to 6 by looking at a graph of the situation:



The actual increase in U* when CBL increasees from 5 to 6 is 90.25 - 79 = 11.25. The derivative of U* with respect to CBL is 12. Mathematically, this means that the slope of the function at CBL = 5 is 12. Think of this as a rate: if CBL were to increase by a little bit, U* would increase at 12 times the increase in CBL. For example, if CBL increased by 1/12, U* would increase by almost exactly 1. But note that as soon as you go any distance above 1, the rate of change (that is, the slope) decreases: the function bends down. So the actual change in U* when CBL increases from 5 to 6 is going to be less than the slope evaluated at CBL=5 predicts.

The point is this: the derivative of a reduced form tells you how much the function changes when an exogenous variable changes by an INFINITESIMALLY SMALL amount. If you have a non-linear function and you want to know the effect of a LARGE change in the exogenous variable (say, from 5 to 6) then DO NOT USE THE DERIVATIVE. It is better to use the Method of Actual Comparison. The derivative tells you the rate of change in the immediate neighborhood of the optimal value.

ASIDE: Most students have a deep respect for calculus. They think calculus is always more powerful and better than "mere numbers." This is WRONG. Calculus is extremely powerful—if you know how to use it, you can quickly establish the direction and magnitude of the change in an optimal value of an endogenous variable for a given shock. But if the question calls for an estimate of a particular, non-infinitesimally sized shock, then calculus may lead you astray. Pay attention to the question at hand.

Understanding Lambda*

Look again at the expression for dU*/dCBL (on page 11): it should look familiar. If you go back to the expressions for the optimal values of the endogenous variables (on page 9), **you will see that the expression for dU*/dCBL exactly matches the expression for λ*.**

This is no accident!

We now have a meaning for the optimal value of lambda: it measures the "tightness" of the constraint in the sense of how much more the maximum value of the objective function would be if the constraint was relaxed a little bit. Remember that lambda* comes in units of the optimum value function per unit of the exogenous limit in the constraint and so is a rate of change concept.

> **The optimal value of the Lagrangean Multiplier is simply the derivative of the optimum value function with respect to the exogenous limit in the constraint. The optimal value of lambda tells you *the rate at which the optimal value of the objective function rises when the constraint relaxes a little bit.* In this case, it tells us how fast Wally's utility would increase if the doctor increased his limit by an infinitesimally small amount.**
>
> **λ* is often known as a "shadow price" in constrained optimization problems, the price that the agent is willing to pay to have the constraint relaxed a little bit.**

Other Applications and Interpretations of Lambda*

• Suppose that a consumer is maximizing utility subject to an income constraint. Then the optimal value of lambda is the extra maximum utility that would be enjoyed if the consumer had another dollar to spend.

λ* is measured in utils/$

• Suppose that a firm is maximizing profits subject to a constraint on factory floor space. Then the optimal value of lambda is the extra maximum profit that could be earned if the firm had one more square foot of floor space.

λ* is measured in $/sq. ft.

• Suppose that a firm is minimizing the costs of producing a given level of output. Then the optimal value of lambda is the additional (or marginal) minimum cost of producing one more unit of output.

λ* is measured in $/unit of output

A Final Comment on λ^*

Every solved constrained optimization problem has an optimal value of λ; it reveals how much the optimum value function changes when you relax the constraint. As such, λ is always a rate, something *per* something else. If you remember that the units of λ^* are "units of the optimum value function *per* unit of the exogenous limit in the constraint," you should be able to interpret correctly any value of λ^*.

λ^* is a ***powerful shortcut***. Often we want to know what would happen to the optimum value function if the constraint was relaxed.

> One way to answer this question is through the Method of Actual Comparison. Solve the problem over and over again for different values of the exogenous limit in the constraint, then see how the optimum value function responds to these discrete changes in the exogenous limit. This will tell you what would happen to the optimum value function if the constraint was relaxed.
>
> Another way is to solve the problem via calculus with the exogenous limit in the constraint as a "letter," instead of a "concrete" number (e.g., total cigars and brandies allowed represented as "CBL" instead of as "5"). Work your way through the solution recipe and Step 5 will generate a optimum value function that contains the variable "exogenous limit in the constraint" (as we showed above, $U^* = 0.25 + 19.5CBL - 0.75CBL^2$). Then take the derivative of the optimum value function with respect to the exogenous limit in the constraint ($dU^*/dCBL = 19.5 - 1.5CBL$) and evaluate it at a given value of the exogenous limit ($dU^*/dCBL = 19.5 - 1.5(5) = 12$). This will tell you what would happen to the optimum value function if the constraint was relaxed.
>
> A third way is to say, "Hey, I know that λ^* reveals how much the optimum value function changes when you relax the constraint." If you know λ^*, it will tell you what would happen to the optimum value function if the constraint was relaxed. That's a ***powerful shortcut***.

An Annoying Detail

It turns out that in this problem, we really need to add two other constraints: C and B must both be constrained to be greater than or equal to 0. Why? Well, it just doesn't make sense to allow consumption of cigars or brandy to be negative. Such <u>inequality</u> constraints are tough to deal with and usually aren't missed, so we won't talk about them much in this course. (Fortunately, it is easy to get Excel to take into account inequality constraints.) If you want to see why an inequality constraint is needed, plug CBL = 0 into the reduced forms for C^*, B^*, λ^*, and U^* on page 9 and think about the resulting values of C^*, B^* and U^*.

ANSWERS

Q: Finish the statement below:

The derivative above shows that C* is linear and increasing in CBL because:

THE DERIVATIVE DOES NOT CONTAIN CBL, HENCE, C* GRAPHED AGAINST CBL IS A STRAIGHT LINE AND THE DERIVATIVE GRAPHED AGAINST CBL IS A HORIZONTAL LINE. WE SAY THAT C* IS LINEAR IN CBL.

THE DERIVATIVE IS POSITIVE (0.75) WHICH MEANS THAT THE C* IS UPWARD SLOPING AND SO MUST BE INCREASING AS CBL INCREASES.

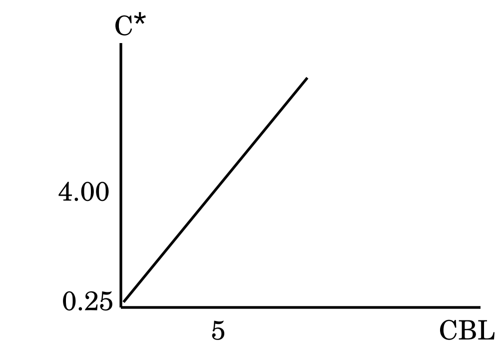
MORE EXPLANATION:

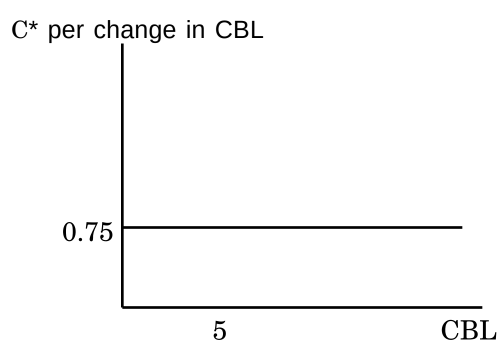
The reduced form for C* is

$$C^* = 0.25 + 0.75CBL$$

and so the derivative is

$$\frac{dC^*}{dCBL} = .75$$





Look at the top graph—it shows C* as an increasing, linear function—no matter the increase in CBL, C* will increase by 0.75-fold.