Chapter 13: Introduction to Equilibrium

Taking Stock:

So far, we have covered one of the two major parts of the Economic Approach. We've explored how the Economic Approach can be applied to **optimization** problems (of both the unconstrained and constrained varieties). This chapter marks the beginning of our analysis of **equilibrium** systems.

Some of the Skills You've Gained:

Throughout the first 12 chapters, you have accumulated a long list of new skills and tools.

- On the computer, you have learned to use Microsoft Excel to do a wide variety of calculations and presentations of results. For example, you have learned to save time by selecting a cell and filling down or right, to avail yourself of relative and absolute references, and to navigate around complicated workbooks. You can draw several different kinds of charts in Excel. You know how to set up and solve problems using Excel's Solver and then do comparative statics

- In mathematics, you have learned about the difference between discrete, Δ, versus infinitesimally small, d, changes (upon which the derivative and calculus are based) and why marginalism (setting the derivative equal to zero) works. You have learned how to solve equations in order to get reduced-forms. You have seen how the optimal value of the Lagrangean Multiplier has a powerful economic interpretation.

Of course, the Excel and mathematics skills you have acquired are a means to the ultimate end—a better understanding of ***ECONOMICS***.

- In economics, we have emphasized a well-organized set of ideas and graphs that are used repeatedly in different problems. All optimization problems must be set up, solved, and then analyzed in terms of comparative statics. Elasticity is a convenient way of representing responsiveness. Graphs are not individual and separate, but belong to a logical taxonomy and, thus viewed, are better understood. Underlying graphs show why a solution is optimal, while presentation graphs display comparative statics analyses.

Before we continue, we need to make sure you are completely comfortable with where we've been and where we're going. Thus, we offer a basic review of the important economic ideas and terms we have learned thus far.

Remember the Big Picture:

The only way to make any sense of the maze of new skills and ideas is to have a *logical, organizedscheme* of what you've done and how those new skills fit into the big picture.

- You must see the separation of exogenous from endogenous variables. It's not a once and for all separation; it depends on the agent's perspective. The same price variable can be endogenous (to a monopolist) and exogenous (from the point of view of a single consumer). The same is true in equilibrium models.

- It must be clear that economists can be either finding an initial optimal solution OR doing comparative statics. These two different questions use different graphs and procedures. The same is true in equilibrium models.

- There are many ways to solve an optimization problem. Marginalism via calculus and Excel's Solver are two of the alternatives. The same is true in equilibriummodels.

The Lesson of the Economic Approach Repeated:

We have stressed that economics interprets an observed behavior as if it were an optimal solution to an underlying optimization problem. ***Changes in observed choices are seen as different optimal responses to changed environmental conditions.*** When an economist sees a changed behavior, he or she does not wonder if the agent "wised up" and found a better solution. Instead, the economist immediately begins to search for the exogenous shock that generated the new optimal solution.

> For example, recall that you learned that an economic explanation of JIT would rely on changes in exogenous carrying and shipping cost shocks. Firms, maximizing profits yesterday without JIT inventory practices, found that maximizing profits today required using JIT.

The same is true in equilibrium systems. Economists are sometimes interested in the shock that caused a change in the equilibrium solution. You shouldn't forget what you have learned thus far because:

Many of the skills and lessons from optimization problems are used when the Economic Approach is applied to equilibrium systems.

An Important Pedagogical Aside:

In the pages that follow, we will introduce you to equilibrium models. It turns out that "equilibrium" is not an easy term to define precisely. There are a wide variety of associated terms and corollary ideas that together form the study of equilibrium systems in economics.

In order to explain the notion of equilibrium, we will spend some time describing ideas that we will not use in the chapters that follow. Our extended discussion of steady-state, for example, is meant to place "equilibrium" in its proper intellectual context. We will also mention advanced tools and methods for the sake of completeness and not discuss them again.

Use this reading to understand the ***idea of equilibrium*** and how it fits into the Economic Approach.

THE *VOCABULARY* OF THE ECONOMIC APPROACH APPLIED TO EQUILIBRIUM SYSTEMS

We begin with the word, "system:"

System:

A collection of interacting forces, inter-related agents, or inter-dependent elements forming a complex whole.

Some Examples of Systems for you to Ponder:

- A rain forest
- NCAA Division I football teams
- Cells in your body
- Consumers and producers of goods and services

In economics, the agents in the system exert forces based on optimizing behavior. We will often use reduced forms or presentation graphs derived from individual optimization problems as the building blocks of an economic model of a system.

For example, an economist might interpret the observed price in a market as the result of the opposing forces exerted by buyers (who bid and drive the price up) and sellers (who compete with each other by offering lower prices). Each buyer and seller is an individual optimizing agent. Collectively, the optimizing responses of these agents constitute a force in the system.

The Economic Analysis of Systems:

Some systems are called "equilibrium systems." These systems have a tendency to establish, by means of the action of the opposing forces of the elements inside the system, a particular set of repeated values for the variables that are determined by the system. Equilibrium is the economist's basic rationale for the order exhibited by decentralized, seemingly chaotic systems.

As with optimization problems, the economist tries to

(1) Set Up the Problem, so that he or she can

(2) Find the Equilibrium Solution to the system.

This is a prelude to the final step of

(3) Comparative Statics—i.e., an exploration of how the equilibrium solution varies when there is an exogenous change, *ceteris paribus*.

Equilibrium and Steady-State:

You would think that a concept as commonly used and discussed as "equilibrium" would have a simple definition that was mutually agreed upon. Surprisingly, however, equilibrium has alternative meanings and no single precise definition.

Equilibrium is not easily defined because a *different perspective will result in a different definition of equilibrium*.

Below, we offer two definitions of equilibrium:

- A position of *rest*; a value of a variable which has *no tendency to change*
  Example: A swinging pendulum has "settled down" and is no longer moving. It is now in equilibrium.

- A state of *balance* between opposing forces or actions
  Example: A scale that measures your weight when you stand on it and either put on or take off lead weights on a bar. It is in equilibrium when it stops fluctuating up and down, remaining balanced in the center—and telling you how much you weigh!

In addition, the system may exhibit a repeated pattern without ever coming to a single point of rest. Such a situation can be called a ***steady-state solution.***

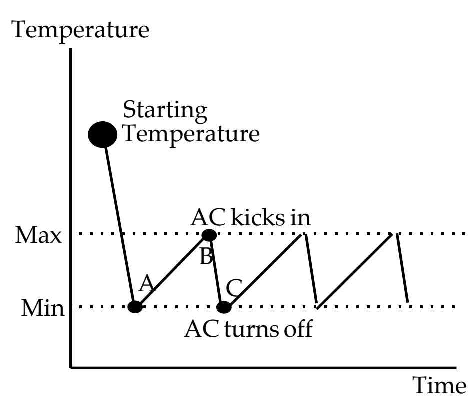
This concept of steady-state, unlike the notion of equilibrium, causes problems because the solution is a series of constantly repeated values instead of merely a single number.

Example: Look at the series:

2, 4, 6, 2, 4, 6, 2, 4, 6 . . .

The steady state solution is 2, 4, 6 (or 4, 6, 2 or 6, 2, 4)—here the phrase "steady state" refers not to a single repeated value, but to a repeated pattern.

Example: Think of how a central air-conditioner works in a room. The ***SERIES*** of values from A to B to C are repeated over and over:



THINKCAREFULLY:

Most students are confused by the Central AC example. It seems to be oscillating. The graph seems to show a cycle—unlike the single resting point of the pendulum or the scale in the two previous examples. As such, there is no single position of "rest"—the temperature is always changing! But the point is that although the solution itself, the series of repeated values, may be a cycle, the values are being consistently revisited over and over again in a regular, established pattern. Central AC works by taking you from any initial temperature directly to the MIN or MAX temperature value. Once there, it follows the "steady-state path" as long as the AC works and all other exogenous variables (including outside temperature) remain constant!

PERSPECTIVE IS CRUCIAL:
As we said above, *perspective* plays a crucial role here. Once in its steady-state path, the temperature obviously is constantly changing from one minute to the next. Seen narrowly, at each instant in time, there is no equilibrium, for there is no single temperature that the room gravitates toward. There is, however, a steady-state solution if the perspective is changed from the temperature at a single moment to the broader point of view of an entire series of temperatures. From this broader perspective, we see a repetitive pattern.

MORE THAN ONE EQUILIBRIUM SOLUTION IS POSSIBLE:
We have just seen that a system may not have an equilibrium solution, but may have a steady-state solution. To further muddy the waters, it is possible for systems to exhibit different solutions whereby one equilibrium or steady-state might emerge under one set of conditions and another, completely different solution might result from a different initial condition.

Do not think of equilibrium only in the narrow, market sense of quantity demanded equals quantity supplied. That is a particular type of equilibrium that is important in the study of economics, but we want to understand the concept first, then apply it to economics.

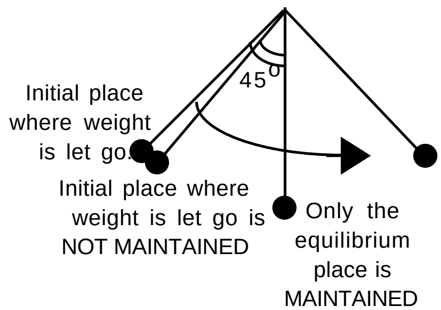
Let's continue this important discussion of equilibrium by considering it's opposite, disequilibrium.

Disequilibrium:

A value of a variable which is NOT its equilibrium value in the sense that the value will be moved to a different value by the forces in the system and will not be consistently revisited

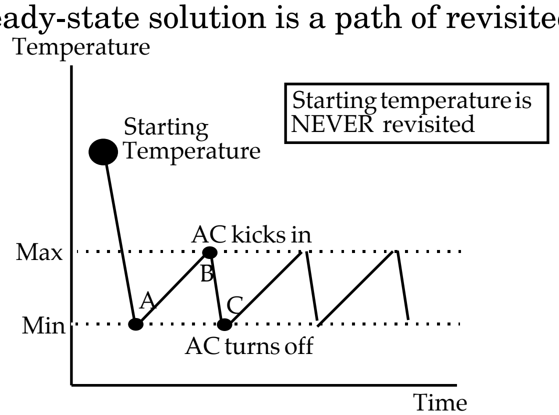
Some Examples to Ponder:

• The Pendulum
If we took a snapshot of a swinging pendulum, the position of the weight at the end of the string would be in disequilibrium. Depending on where in the swing it was, it could be continuing upwards or coming down. In either case, forces (gravity and friction) are acting on the weight's position to change it in such a way that the position will not be maintained:



But just to make sure you understand the concept of steady-state, we offer for your consideration a pendulum in a vacuum. Absent the forces of gravity and friction, the pendulum will swing from the initial place where it was let go, exactly to the other side, and then back to the initial place. It will do this forever. In a vacuum, the steady-state solution is that path that it continues to repeat over and over again.

• The Temperature in a Centrally Air-Conditioned Room
As we saw above, the steady-state solution is a path of revisited values:



The starting temperature is a disequilibrium value from the perspective of the repeated cycle because the starting temperature will NOT be revisited. Temperatures from A to B to C, on the other hand, are part of a steady-state path, because they will be repeated.

Important Adjectives Attached to Equilibrium

ASIDE: While the comments below also apply to steady-state solutions (defined as a series of repeated values), we will concentrate on equilibrium (defined as a single point solution) in the definitions that follow. We explained the idea of a steady-state only as a means to improve your understanding of the idea of equilibrium. Having done so, we now focus on equilibrium (with only a few comments about analogous steady-state applications).

Economists not only worry about whether a particular value is in or out of equilibrium, they also study the type of equilibration process, or how the system moves over time. This study gives rise to a series of adjectives that describe the kinds of equilibrium.

Equilibria can be either:
Stable (Convergent) or ***Unstable*** (Divergent)

The fundamental question here is:

Will the system return to its equilibrium solution if perturbed?

If yes, it is a stable (convergent) equilibrium solution.
If no, it is an unstable (divergent) equilibrium solution.

The pendulum and central a/c are examples of stable systems. A coin stood on its end is an example of an unstable equilibrium. Once you carefully stand the coin on its end, absent any perturbations, it will stay that way. But if you tap it (hard enough), it will tip over and will not stand itself back up!

Equilibration processes can be either:
Oscillatory versus ***Non-Oscillatory***

The fundamental question here is:

How will it move toward or away from its equilibrium solution?

If it moves in a cycle—up and down, over and under, past the equilibrium solution, then back—it is oscillatory.

If it goes directly to the equilibrium solution—without overshooting it—it is non-oscillatory.

The pendulum is an example of an oscillatory equilibration process. It swings by the equilibrium solution, getting closer and closer every time.

Most students would describe the central a/c example as oscillatory for the temperature is moving in a cycle. But do not be confused. Oscillatory or non-oscillatory refers not to the solution itself, but *how we get there*. From any starting temperature, the a/c takes you immediately to the steady-state solution. It does not swing you high and low, gradually converging to the steady-state solution. Thus, the a/c example exhibits non-oscillatory convergence to the steady-state solution.

The Types of Equilibrium:

A table of the different kinds of equilibrium looks like this:

Stable Equilibrium/Steady-State		Unstable Equilibrium/Steady-State	
Oscillatory Convergence	Non-Oscillatory Convergence	Oscillatory Divergence	Non-Oscillatory Divergence

Having discussed SYSTEMS and the notions of EQUILIBRIUM and STEADY-STATE, we now turn our attention to the types of VARIABLES in every equilibrium model of a system.

Endogenous versus Exogenous Variables:

Endogenous variables are determined from *within* the system by the interaction of the forces in the system; exogenous variables are given and fixed (they do not change at all, no matter what the system does).

In a market system, price is an endogenous variable. It is pushed and prodded by the forces within the system. At that value or set of values when (or if) price begins to repeat itself over and over, we have the equilibrium solution. Even there, the value of the variable is being determined by the system.

The slope of the demand curve, on the other hand, is exogenous. It is determined by the particular characteristics of the consumers. It remains the same during the analysis of the equilibrium solution.

EquilibriumNotation:

An "e" is attached to the equilibrium value of an endogenous variable.

Thus, we now have two different kinds of reduced forms, where y is an endogenous variable and x an exogenous variable:

Optimization: $y^*=f(x)$

Equilibrium: $y_e=f(x)$

Equilibrium versus Optimization:
Clearly, a distinguishing characteristic of an optimization problem versus an equilibrium system is the way the endogenous variable is interpreted.

> In optimization, the agent exercises direct control over the endogenous variable, assigning it a particular (optimal) value.
>
> In equilibrium, elements within the system exert opposing forces which bring the endogenous variable to its equilibrium solution. *No one agent sets the value of the endogenous variable; it is determined by the forces in the system.*

This brings up a crucial point:

e does not necessarily equal *

A persistent source of error throughout the history of economics has been the equating of equilibrium with optimal solutions. ***There is nothing inherently or automatically optimal in an equilibrium solution.***

> It is quite clear that a system could settle down at a place that was distinctly suboptimal. For example, a monopolist in a market leads to an equilibrium solution. At the monopoly price and output, there is no shortage or surplus. There is no tendency to change. We have an equilibrium solution. Most students can explain, however, that from society's perspective the P, Q monopoly solution is inefficient because too little of the monopolist's product is produced. The equilibrium solution is NOT the socially optimal solution in the case of monopoly.
>
> In fact, this point can be extended to explain that an optimal solution depends on the perspective of the objective function. The price, quantity combination chosen by the monopolist is optimal because profits are maximized. From a societal resource allocation perspective, however, the monopoly's price, quantity combination is inefficient. Of course, the monopolist is quite happy and from the monopoly's point of view, the solution is optimal. This should make it clear that ***There is nothing automatically GOOD about an optimal solution.*** It all depends on the perspective from which the optimization problem is analyzed.

Thus, although an equilibrium solution ***may*** equal an optimal solution, it takes much work and careful analysis to find an optimal solution and then compare the equilibrium to the optimal solution. ***You cannot and should not immediately equate equilibrium and efficiency.*** They are separate ideas.

We now know that equilibrium systems are composed of endogenous and exogenous variables, but how is the equilibrium model put together?

Structural Equations and Equilibrium Conditions:

The structural equations are the economist's interpretation of the forces in the system in a mathematical form. These equations represent the separate pieces of the system and show how the forces interact with each other.

> For example, demand and supply functions would be the structural equations in an equilibrium model of a market.

The equilibrium condition(s) describe what must be true for the system to be in equilibrium. Usually, the equilibrium condition sets the structural equations equal to each other or somehow combines them.

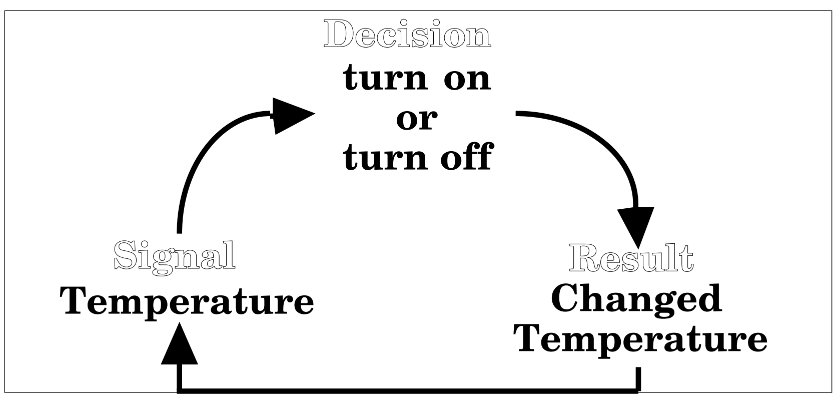
> For example, the equilibrium condition in the market model would be that quantity demanded equaled quantity supplied. At that point, the system is in equilibrium.

FeedbackMechanism:

Every equilibrium system has a feedback mechanism or "loop" in it. The feedback mechanism describes how the system alters the endogenous variables as they pass through the loop over and over again. There are three key factors in the feedback mechanism: ***1) Signal, 2) Decision, and 3) Result***.

Let's examine the three key factors in our steady-state A/C example. The air-conditioner's thermostat measures the temperature in the room. The temperature sends a ***SIGNAL*** to the thermostat. The thermostat then makes a ***DECISION***—turn on if temperature is too high (to cool the room); turn off if temperature is low enough. The ***RESULT*** is a change in temperature. Temperature is the endogenous variable that is determined by the system.

Temperature constantly passes through this feedback mechanism and is kept on its steady-state track by virtue of the ***SIGNALS, DECISIONS*** and ***RESULTS*** of the feedback mechanism.



When analyzing equilibrium (or steady-state) systems, keep your eye out for the feedback mechanism and how the endogenous variable gets "ground out" by the forces in the system. As we did in our study of optimization, we will try to point out the repeated logic and common elements inherent in all equilibrium models.

Finally, we tackle the most difficult characteristic of optimization and equilibrium systems—the role of time in the analysis.

Statics versus Dynamics:

In economics, Statics implies "timeless analysis." The passage of time, in the sense of how events unfold one after another, is completely suppressed.

> In Static Optimization (which you have been doing thus far), we have the agent choose values of endogenous variables to optimize a specific objective function, *absent any element of time*. This is static analysis.
>
> In Static Equilibrium analysis, we find the values of the endogenous variables that will satisfy some specified equilibrium condition, *ignoring how you get from one equilibrium solution to another*. This is static analysis.

Have you ever thought about what "static" in "Comparative Statics" connoted? The definition of comparative statics is the response of y^* or y_e to a change (be it Δ or d) in an exogenous variable, *ceteris paribus*. We have learned to use reduced forms as equations or presentation graphs to show how an optimal or equilibrium value responds to exogenous shocks. But reduced form, comparative statics analysis says nothing about how the new solution is reached—it's an instantaneous (timeless) jump from initial to new.

LESSON: The point of the word "static" in "comparative statics" is that THERE IS ***NO*** ANALYSIS OF HOW THE NEW OPTIMAL OR EQUILIBRIUM SOLUTION IS REACHED. All we do is compare the initial and new solutions—ignoring completely how we travel from one to the other.

ASIDE on Comparative Statics Analysis in Optimization versus Equilibrium:

We have seen that:

> Optimization presentation graphs connect highest (or lowest, if minimizing) points for given values of an exogenous variable. To be off the graph is to be off a ***hilltop*** and, therefore, the optimizing agent will not be off the reduced form.

We will see that:

> Equilibrium presentation graphs connect equilibrium points for given values of an exogenous variable. To be off the graph is to be in disequilibrium and the value of the endogenous variable will be drawn toward the line or curve. The line or curve is like a ***magnet***. IS and LM curves in macro models are magnets!

Having pointed out that Statics is timeless analysis, Dynamics, on the other hand, means "analysis over time." The object of Dynamic analysis is to trace and study the time paths of the variables. DYNAMIC ANALYSIS IS MUCH MORE DIFFICULT THAN STATIC ANALYSIS.

In Dynamic Optimization, we would choose values of endogenous variables *in each time period* to optimize a specific objective function. Such a study would lead us to topics like: Optimal control theory, Hamiltonians, and the calculus of variations.
We did NOT and will NOT cover this.

In equilibrium models, there are TWO definitions of Dynamic Analysis:

Equilibrium Definition 1—find the time path of the steady-state values of the endogenous variables (as in the a/c example).
We will NOT cover this.

Attainability and Equilibration Process
Equilibrium Definition 2—determine whether, given sufficient time, the endogenous variables will settle down, converge to their equilibrium values, how, and at what speed. This is an analysis of the *equilibration process*.
We WILL cover this.

Note how there is no comparable question to this in optimization. We NEVER think of the optimizing agent as groping his or her way to some hilltop. The agent immediately goes there. Not so in equilibrium models. Economists are interested in how systems equilibrate and they study equilibration processes.

Since we are going to study dynamics in the setting of equilibrium models, we must introduce different types of time and some notation:

Dating of the variables

Continuous time—time is infinitely divisible and changes occur at *points* in time
Differential equations; integral calculus
We will NOT cover this.

Discrete time—time comes in periods of finite length and changes occur once within a *period* of time
Difference equations—an equation that describes how a variable changes over time
We WILL cover this.

An Example of a Difference Equation:

$$\Delta y_t = y_t - y_{t-1}$$

The equation says, "The change in y at time t is equal to the value of y in time t minus the value of y in time t minus one."
Suppose that y was 10, 11, and 14 in year 1901, 1902, 1903. Then, the equation says that $\Delta y_{1902} = 1$ while $\Delta y_{1903} = 3$.

Graphing variables and solutions

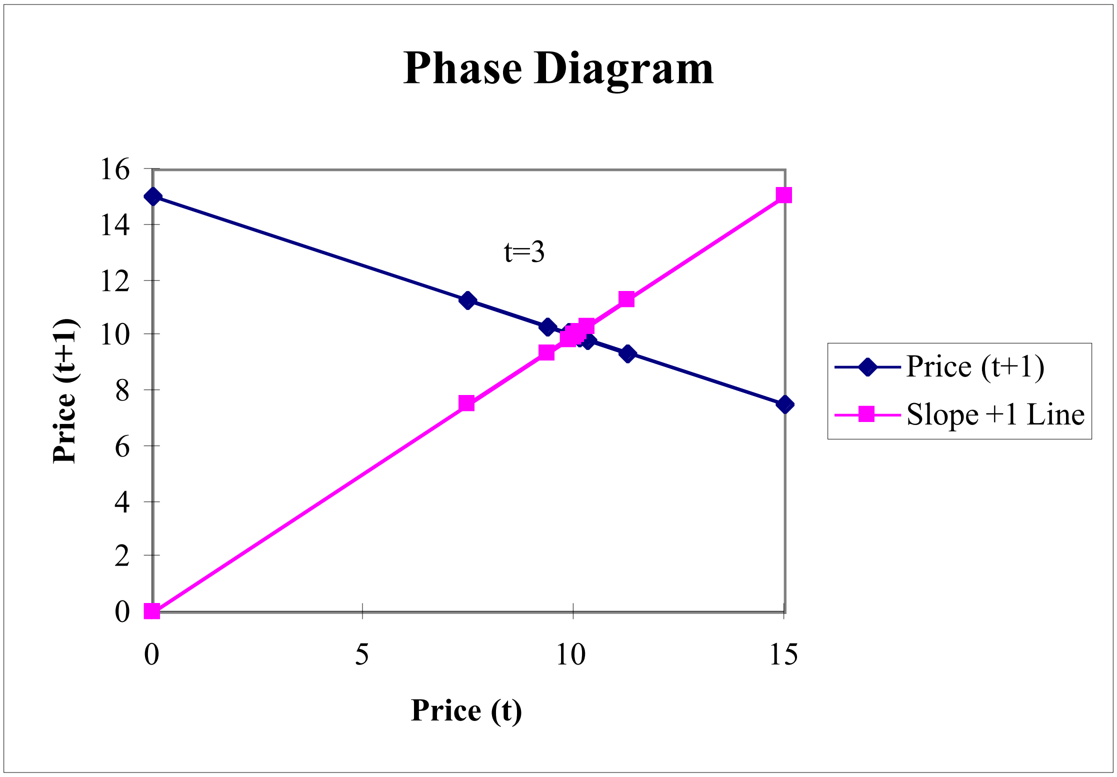
Phase diagram—plot y_t as a function of y_{t-1}

"Graphical iteration"

A way to explore the equilibration process and find an equilibrium solution.

We WILL cover this.

An Example of a Phase Diagram:



What Lies Ahead?

An analysis of equilibrium systems that relies on the recipe of the Economic Approach:

1) Set Up the Problem

a) ***Endogenous Variables***
b) ***Exogenous Variables***
c) Structural Equations
d)EquilibriumConditions

These four pieces make up the equilibrium system and contain the feedback mechanism.

2) Find the Equilibrium Solution

We will use mathematics and Excel.
We will explore the equilibration process—i.e., how the solution is reached.

3) Comparative Statics

We will study the following equilibrium models:

1) Equilibrium Profit in an Entry/Exit Model
2) Equilibrium Output in a Macro Model
3) Equilibrium Rate of Return in an Educational Choice Model
4) Equilibrium Allocation in a Pollution Permits Model
   This last model will enable a discussion of optimization versus equilibrium.

We have left the world of optimization problems and entered that of equilibrium systems. Please be aware of this division in the framework of the Economic Approach. In particular, ***do NOT make the common mistake that an equilibrium solution is automatically an optimal one.*** Equilibrium and optimality are separate ideas. The "e" versus the "*" notation on an endogenous variable makes all the difference in the world. We will repeat this at every opportunity.

You will continue to improve your existing Excel and mathematics skills and learn new ones. These tools will enable you to learn the economics of equilibrium systems. We hope you enjoy the examples and concepts.