# Chapter 17: Equilibrium in a Macroeconomic Model with Microsoft Excel

## Purpose of this Lab:

This lab uses Microsoft Excel to analyze the Simple Keynesian Model we discussed in Chapter 16. It assumes familiarity with the model and builds upon the lessons of the previous chapter.

The overall organization of the lab is straightforward:

| | |
|---|---|
| INITIAL: | Setting up the Problem and Finding the Equilibrium Solution |
| EQ PROCESS: | Exploring the Equilibration Process |
| COMP STATICS: | Comparative Statics |

## Special emphasis is placed on four ideas:

• gaining a richer understanding of the phase diagram by seeing how it looks in this example

• interpreting the presentation graph and "points off the line or curve"

• understanding the Method of Actual Comparison (Δ) versus the Method of the Reduced Form via calculus (d)

• understanding the difference between point-to-point comparisons and approximation via the derivative

## More About this Lab:

This lab is quite long. It requires patience and attention to detail. It covers all of the important ideas of the Economic Approach as applied to equilibrium systems.

There are nine (9) questions in all.

Questions 1, 2, 3, 5, 6, and 7 are to be answered in the Excel workbook, C17lab.xls.
The last page of this document is where you provide answers to Questions 4, 8, and 9.

It is imperative that you have read and have available C16Read.pdf because that chapter contains valuable information about the workings of the Simple Keynesian Model that you are analyzing here.

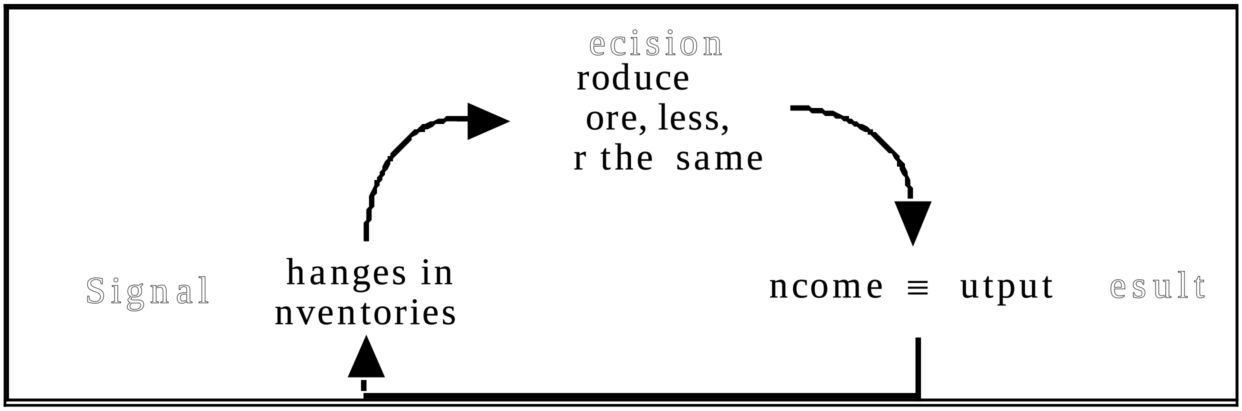
Getting Started: Launch Excel and open the file called C17Lab.xls.

*Good luck.*

## INITIAL SOLUTION

• To be read BEFORE working with the sheet INITIAL in C17Lab.xls

Introduction:
Having read C16Read.pdf, you know the basic elements in the Simple Keynesian Model. In a picture, the "feedback mechanism" looks like this:




Given a level of income, consumers decide how much to buy. Consumer spending is added to investment spending by firms and government spending by the government to determine aggregate demand. This aggregate demand is then fulfilled. Now, there are three possible cases:

1) AD greater than actual GDP produced that year
   In this case, the excess aggregate demand over the amount of goods and services produced that year is fulfilled out of inventories which are, obviously, depleted.
2) AD less than actual GDP produced that year
   In this case, the goods and services produced that year more than meet the aggregate demand and the left over stuff is added to inventories, which are, obviously, augmented.
3) AD exactly equals actual GDP produced that year
   In this case, the aggregate demand is fulfilled exactly out of the amount of goods and services produced that year without any changes to existing inventories.

If the first case, firms respond to the *signal* of depleted inventories by making a *decision* to increase production. They hire more workers to make that additional output and, in turn, this *results* in higher income to the consumers/workers group. Consumption increases. This leads to a *higher* aggregate demand which is compared to actual GDP produced that year.

If the second case, firms respond to the *signal* of augmented inventories by making a *decision* to decrease production. They fire or lay off workers to because they are making less output and, in turn, this *results* in lower income to the consumers/workers group. Consumption decreases. This leads to a *lower* aggregate demand which is compared to actual GDP produced that year.

If the third case, firms respond to the *signal* of unchanged inventories by making a *decision* to not change production. They keep the same number of workers to make that same output and, in turn, this *results* in the same income to the consumers/workers group. Consumption is unchanged. This leads to the *same* aggregate demand which is compared to actual GDP produced that year. This is the equilibrium solution.

C16Read.pdf covered a more formal mathematical presentation of the Simple Keynesian Model. After completing Step 1) Set Up the Problem, in which the endogenous and exogenous variables were described and the structural equations and equilibrium condition was presented, the Simple Keynesian Model looked like this:

Structural Equations:

$$C = c_0 + MPC(Y - t_0 Y)$$

$$I = I_0$$

$$G = G_0$$

$$AD = C + I + G$$

and
Equilibrium Condition:

$$Y = AD$$

In C16Read.pdf, we showed how a pencil and paper solution could be found to this general problem and to a concrete problem (where the exogenous variables are given numerical values):

Suppose:

$$C = 200 + .8(Y - .0625Y)$$

$$I = 200$$

$$G = 100$$

then (after some algebraic manipulation), $Y_e$ = \$2000.

| Go to C17Lab.xls now in order to see how Excel can be used to find the equilibrium solution. |
|---|

# THE EQUILIBRATION PROCESS

• To be read BEFORE working with the sheet EQ PROCESS in C17Lab.xls

Introduction:

We have seen in the Profit Equilibration example and the discussion of the Simple Keynesian Model that the type of equilibration process can be analyzed by several different kinds of graphs.

Graphs of the endogenous variable over time have the virtue of being easy to read and, usually, of quickly showing the type of equilibration. They suffer, however, from the fact that every system is different and so it is difficult to compare equilibration across models by looking at simply how the endogenous variable travels over time.

Phase diagrams have a powerful consistency that can be exploited to quickly answer the three kinds of questions about the equilibration process:

- Is the equilibrium stable?
- How is it reached?
- How fast is it reached?

We offer the following from C15Lab.pdf:
The big advantage of phase diagrams lies in their consistency. Different models will show different types of pictures when the endogenous variable over time or one endogenous variable as a function of another endogenous variable graphs are used. They'll have different break points for the seven regions. The phase diagram, on the other hand, remains the same.

The LESSON of phase diagrams is that the intersection of the phase line with the slope of +1 line shows the equilibrium solution(s) AND that the SIGN (> or < 0) and MAGNITUDE (numerical value) immediately reveals the type and the speed of the equilibration process.

This is a lesson worth remembering and one that we will use in other equilibrium models.

From your work in C15Lab.pdf, you discovered the following:

• Equilibrium occurs wherever the phase line intersects the slope +1 line

• The type of equilibration process breaks down into seven distinct regions that look like this:

| Slope of phase line | Type of Equilibration Process |
|---|---|
| Less than -1 | Oscillating divergence |
| Equal to -1 | Uniform oscillation |
| Between -1 and 0 | Oscillating convergence |
| Equal to 0 | Instantaneous convergence |
| Between 0 and +1 | Direct convergence |
| Equal to +1 | No movement |
| Greater than +1 | Direct non-convergence |

• The speed toward equilibrium increases as the phase line approaches 0

As applied to the Simple Keynesian Model, we asserted that the equilibration process looks like this:

$$Y_{t+1} = Y_t + \Delta Y_{t+1}$$
$$\text{where } \Delta Y_{t+1} = \text{rho} * (Y_t - AD_t)$$

Basically, the equation above says that the change (Δ) in output in the next period is some proportion (rho) of the level of the difference between output and aggregate demand in the current period. Rho is some exogenous variable whose value depends on the system you're looking at. Different values of rho will generate different kinds of equilibration.

That's the story on a general level. Now, let's slowly and carefully try to figure out more precisely what's going on by walking through some numbers.

Suppose the initial level of the value of actual output (GDP) is Y = $400. Then, our structural equations tell us that aggregate demand is $800 (since 200 + 0.8*(500 - 0.0625*500) + 200 + 100 = $800). Inventories are depleted by $400 (since $400 - $800 = *negative* $400 where the the negative means "depletion" in inventories). In response to the fall in inventories, more is produced next time. *By how much does output rise?* This is crucial to a specific description of the equilibration process.

Suppose that, because of the speed with which firms respond to the depleted inventories, output increases by TWICE the depletion of inventories so that *the value of rho is - 2*. According to our description of the equilibration process, the change in Y in the next period will be

$$\Delta Y(t+1) = - 2 \text{ x } - 400 = \$800$$

so that actual GDP next period will be $1200 since that's equal to $400 + $800.

Let's see what happens the next period. With Y=$1200, AD=$1400, so that inventories are depleted by $200, and output rises by $400 to $1600.

Now, with actual GDP = $1600, the value of output next period will be $1800 and after that it will be $1900. And so on, until the value of output "settles down", i.e., reaches an equilibrium solution.

Figuring out the path of the endogenous variable output over time is one way of describing the equilibration process. We will do this in a moment.

## More on rho:

The key to the equilibration process is rho. Notice that we made rho < 0. Why? Because output *falls* when Y - AD is *greater than* zero; and output rises when Y - AD is less than 0. If the economic argument that output and the gap between Y and AD are inversely related holds true, rho will be negative. The *size and sign* of the exogenous variable “rho” describes the speed and type of the equilibration process.

## More on Excel:

If you are thinking that there’s no way you are going to chug through repeated calculations of the sort found on the previous page as we slowly worked from Y = $400 to $1200 to $1600, don’t worry, YOU WON’T HAVE TO!

This type of mindless calculation is what computers are exceptionally good at!!!

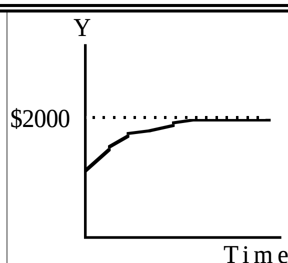
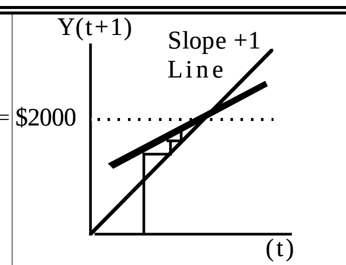
> Return to Excel now and go to the sheet called EQ PROCESS. It's time to put these lessons to work!

WELCOME BACK!

*QUESTION 4:*
Now that you have successfully drawn a phase diagram for rho = - 2, use that very same chart you just created to fill in the very last page of this handout.

Be smart! Remember the lesson of the phase diagram . . .

Provide not only an example numerical value for the value of rho and the slope of the phase line corresponding to a particular type of equilibration process, *but also their ranges*. So, in the case we've done for you, we give the following:

| | | | | |
|---|---|---|---|---|
| $-4 < \rho < 0$<br><br>$\rho = -2$ | Non-Oscillatory (Direct) Convergence |  | 0 < Slope of phase line < +1<br><br>Slope of phase line = 0.5 |  |

We give you an example value of rho (-2) and the range for rho that corresponds to non-oscillatory (direct) convergence; similarly, we provide an example slope and the range of the slope of the phase line. Please do the same.

> Go to the next page to finish this lab by analyzing the comparative statics properties of the Simple Keynesian Model.

# COMPARATIVE STATICS

• To be read BEFORE working with the sheet COMP STATICS in C17Lab.xls

## Introduction:

You are quite familiar now with the phrase "comparative statics." By comparing an initial solution to a new solution (brought about by a shock to an exogenous variable), economists can make "if-then" statements that, it is hoped, can be tested empirically. These "if exogenous shock then this change in an equilibrium or optimal value of an endogenous variable" predictions can be qualitative (indicating direction only) or quantitative (which includes not only the direction, but also the amount of the response in the endogenous variable). In order to make quantitative "if-then" statements comparable, economists utilize *elasticity* to report not the change in endogenous variable for a given change in the exogenous variable, but the *percentage* change in endogenous variable for a given *percentage* change in the exogenous variable.

Not only does the student have to maneuver through the slope versus elasticity waters, he or she must also confront the issue of whether the response of the endogenous variable proceeds in a simple, linear fashion or whether the relationship is a more complicated non-linear one.

Comparative statics is the final step in the application of the Economic Approach to equilibrium models. Here we are interested in how equilibrium values of the system respond to shocks. We do not ask how or if we get to equilibrium. Instead, we assume that we get there—fast—and, thus, we merely compare initial equilibrium values to new equilibrium values.

*We offer below another practice run through these issues. Try to see the repeated pattern and take the time to carefully follow the logic of the analysis.*

As always, comparative statics can be done by the Method of Actual Comparison or the Method of the Reduced Form. Once you've chosen a method, you may have an additional decision to make. If you opted for the Method of Reduced Form, you can make either point-to-point comparisons or use a shortcut, by taking a derivative and approximating the answer. On the other hand, if you've chosen the Method of Actual Comparison, you must then make point-to-point comparisons.

## Method of Actual Comparison:

For the Simple Keynesian Model, suppose that government spending, G, increased to $200. This is the shock. Our task is to find the new equilibrium and then compare the initial equilibrium (Ye=2000) to the new equilibrium.

We can find the new equilibrium either by pencil and paper or by using the computer (cells and graphs or Solver). In C16Read.pdf, we showed that $Y_e$ rises to $2400 from $2000 in response to an increase in G of $100. The problem we consider here is a repeat of the one in the C16Read.pdf reading. You can look back at the reading if you want to see the pencil and paper solution again.

Return to Excel now and go to the sheet called COMP STATICS. Use Excel's Solver and the Comparative Statics Wizard to find how G affects $Y_e$ and how t affects $Y_e$. You will make point-to-point comparisons of initial and new values of equilibrium output. You will discuss what "off the line" means in an equilibrium presentation graph and work again with elasticity.

When you return . . .

You have just used Excel's Solver and the Comparative Statics Wizard to complete an exploration of how Equilibrium Output varies with Government Spending and with the tax rate, *ceteris paribus*. By explicitly picking discrete values of G or t and finding the corresponding equilibrium level of output, you have been doing the Method of Actual Comparison.

$\Delta G$:
You have found that \$100 increases in G ($\Delta G$) lead to the *same* \$400 increases in $Y_e$ ($\Delta Y_e$) every time. In mathematical language, we have $\Delta Y_e/\Delta G = 4$. *This is true no matter the value of G and* $\Delta G$. In mathematical language, that means $\Delta Y_e/\Delta G = dY_e/dG = 4$. From any G, any size change in G will lead to a four-fold change in $Y_e$. From G = \$1000, if you increase G by \$200, then $\Delta Y_e$ = \$800 (and the new $Y_e$ = \$1,800).

Notice how your presentation graph of $Y_e = f(G)$ is a straight line. That $Y_e$ is linear in G means that $Y_e = f(G)$ is a line with some intercept and a constant slope. The constant slope means that $\Delta Y_e/\Delta G$ = constant (4, in this example). *This is true no matter the value of G and ΔG—infinitesimally small (d) or finitely, arbitrarily large (Δ) changes have the same four-fold effect upon* $Y_e$.

$\Delta t$:
You have also found that 1% point increases in t ($\Delta t$) lead to *different* decreases in $Y_e$ ($\Delta Y_e$) depending upon the initial value of t. In mathematical language, that's $\Delta Y_e/\Delta t$ = some negative number that depends upon the starting value of t. *The* $\Delta Y_e$ *for a given* $\Delta t$ *is always changing.*

Notice how your presentation graph of $Y_e = f(t)$ is a curve. $Y_e$ is non-linear in t means that $Y_e = f(t)$ is a curve with a changing slope. The changing slope means that $\Delta Y_e/\Delta t$ = some changing value that depends upon the value of t. *The* $\Delta Y_e$ *for a given* $\Delta t$ *is always changing.*

## Method of the Reduced Form (MRF):

The other way to do comparative statics is to solve the model for the endogenous variables as a function of exogenous variables alone, then take the appropriate derivative.

This is demonstrated in the C16Read.pdf reading. There, it is shown that from the structural equations and equilibrium condition, we can get the reduced form (Equation 1)

$$Y_0^e = \frac{c_0 + I_0 + G_0}{1 - MPC_0(1 - t_0)}$$

The zero subscripts are a way to track the equilibrium value of the endogenous variable, $Y_e$, as a function of the five exogenous variables. The reduced form expression above says that the configuration of the exogenous variables at their initial values (denoted by the 0 subscript) will determine an initial equilibrium value of $Y_o^e$.

The Method of the Reduced Form opens up two avenues for assessing the impact of a shock to an exogenous variable. You can either make point-to-point comparisons or you can approximate the impact by taking a derivative.

### Point to point comparison

The point to point comparison using the reduced form is simple: just substitute in values for the exogenous variables we are holding constant and then values for $G_1$, $G_2$, etc. and track the corresponding $Y_1^e$, $Y_2^e$, etc. The change in $Y^e$ for a change in G is then simply $Y_2^e$- $Y_1^e$ divided by $G_2$ - $G_1$.

### The derivative approximation

Instead of the repeated calculations required by the point to point comparison strategy, we can simply take the derivative of the reduced-form expression to see how the equilibrium value behaves. THIS IS A POWERFUL SHORTCUT.

A derivative, dy/dx, can always be used to investigate how a function, y, varies with x. It's sign and magnitude provide information about the relationship of the "y variable" in dy as the "x variable" in dx changes.

The rate of change in the endogenous variable, equilibrium Y, given an infinitesimally small change in G is $dY^e/dG$. You can use this rate of change to *approximate* the effect on $Y^e$ from a finite, non-infinitesimal change in G. We'll show you how with an example.

Understanding How MAC and MRF are Related via an Example:

We know that $dY_e/dG = 4$ (from C16Read.pdf) and this conforms perfectly with our Method of Actual Comparison results. $dY_e/dG = 4$ says that an infinitesimal change in G will give rise to a 4-fold change (in the same direction) in $Y_e$. Thus, if we increase G by .00001, we'll increase $Y_e$ by .00004. That's exactly what our Solver and Comparative Statics Wizard work showed.

The tricky part in the relationship of the two ways to do comparative statics comes in when we find $dY_e/dt$. C16Read.pdf says that at t=6.25% (and $c_0$=200, $I_0$=200, $G_0$=100, and $MPC_0$=0.8), $dY_e/dt = -6400$. That means that a 1% point (0.01) increase in t to t=7.25% should lead to a \$64 decrease in Ye. *BUT IT DOESN'T!!!*

Your work in Excel showed that when t=7.25%, $Y_e$ = \$1938 (or \$1937.984 for the very careful). This result says that a 1% point increase in t (from 6.25% to 7.25%) leads to a \$62 decrease in $Y_e$ (from \$2000 to \$1938).

The last two questions are designed to help you in:

• understanding the Method of Actual Comparison (Δ) versus the Method of the Reduced Form via calculus (d)

• understanding the difference between point-to-point comparisons and approximation via the derivative

*Question 8:* On the back of the C17 Lab Answers sheet, EXPLAIN CAREFULLY WHY the **derivative approximation yields exactly the same answer as the point to point comparison** FOR CHANGES IN GOVERNMENT SPENDING, BUT NOT FOR CHANGES IN THE TAX RATE.

Answering that "One is linear and the other isn't," really isn't enough. Take the time to provide a real answer that demonstrates your complete mastery of this important question.

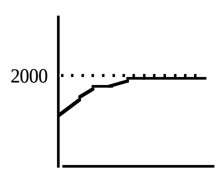
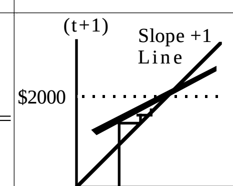
*Question 9:* On the back of the C17 Lab Answers sheet, FOR THE QUESTION OF THE EFFECT ON EQUILIBRIUM OUTPUT FROM A 1% POINT INCREASE IN THE TAX RATE, WHICH METHOD IS BETTER? WHY?

These two questions will show whether or not you really understand the two different approaches to comparative statics. We have been here before and you have access to your work and our answers. We emphasize that this is a crucial part of understanding the Economic Approach. It deserves and is worth careful, concentrated effort.

We hope you will have the opportunity to explore the C17LabA.xls answer workbook and C17LabA.pdf answer key to make sure you understand these concepts.

NAME ________________________________

C17Lab Answers

| Value of Rho (ρ) | Type of Equilibration | Picture of Equilibration Process over Time | Value of Slope of Phase Line | Picture of Equilibration Process with Phase Line |
|---|---|---|---|---|
| | Oscillatory Divergence | | | |
| | Uniform Oscillation | | | |
| | Oscillatory Convergence | | | |
| | Instantaneous Convergence | | | |
| $-4 < \rho < 0$<br>$\rho = -2$ | Non-Oscillatory (Direct) Convergence |  | 0 < Slope of phase line < +1<br>Slope of phase line = 0.5 |  |
| | No Movement | | | |
| | Non-Oscillatory (Direct) Divergence | | | |