Chapter 11: Two-Variable Constrained Optimization via Excel's Solver

GOALS OF THIS CHAPTER:

- To understand CONTOUR MAPS in the constrained optimization case
- To use Excel to solve constrained optimization problems
- To use Excel to find λ^*
- To understand exactly what λ^* means and what it's good for

MORE ABOUT THE LAB:

The Problem:

In this lab you will learn how to use Microsoft Excel to solve the constrained optimization problem we looked at in the last chapter in which Wally chooses utility-maximizing quantities of cigars and brandy subject to a doctor's restrictions.

Contour Maps:

We will see how the advantages of contour maps for two-variable unconstrained optimization problems carry over to constrained optimization problems. Contour maps are the graph of choice for multivariable optimization problems simply because they focus attention on the optimal solution in a way that more complicated 3D graphs cannot.

Excel's Solver for Constrained Optimization Problems:

We have already solved unconstrained optimization problems with Solver and we've seen, but have not used, the constraint section of the Solver dialog box. It is really quite easy to incorporate constrained optimization problems into Excel's Solver.

In addition, we have solved the cigar/brandy problem via calculus so we are familiar with the details and solutions. Not surprisingly, Excel's answers are the same.

Using Excel to find λ^*:

We will extend our knowledge of Excel by asking it to report the value of λ^*. We continue to offer several ways of doing things. You should NOT think of this as a "silver bullet" in the heart of calculus—something like "Great! I'll never have to do calculus again!" Instead, it's another weapon the arsenal—if you have enough alternative solution methods, no problem can stump you.

Understanding λ^*:

In perhaps the most difficult part of the entire course, we will also ask you to think and talk about the interpretation of λ^* (lambda*), the optimal value of the Lagrangean Multiplier from the calculus version of the problem, in the context of the Excel solution to the cigars and brandy problem.

Let's take a few minutes to quickly summarize what lambda* is all about:

When you learned how to solve a constrained maximization problem using calculus, you created a new variable called lambda. Lambda, the Lagrangean Multiplier, is a TOOL which allows you to CONVERT the constrained problem into an UNCONSTRAINED one. In addition, the optimal value of lambda (lambda*) also has a very nice interpretation: lambda* is the rate of increase in the optimal value of the objective function (or U* in the cigar/brandy problem) when the constraint is relaxed by an infinitesimally small amount.

The actual value of the change in the optimal value of the objective function can be calculated in two different ways.

> (1) Via calculus, dU*/d"exogenous limit" is based on the notion of an INFINITESIMAL CHANGE in the constraint. The derivative dU*/d"exogenous limit" is EXACTLY EQUAL to lambda*.
>
> (2) In today's lab we will ask you to calculate the change in the optimal value of the objective function using LARGER, DISCRETE CHANGES in the constraint. The notation for this is ΔU*/Δ"exogenous limit"—where the "Δ" represents a finite, arbitrarily large change instead of the infinitesimally small change that is the hallmark of "d" (the derivative).

It should not be surprising to find that ΔU*/Δconstraint ≠ lambda* when the reduced form for U* is a non-linear function of the exogenously set constraint limit. This is just another case of the difference between a DISCRETE (Δ) change and an INFINITESIMAL (d) change that you have encountered many times before.

THE LESSON ABOUT LAMBDA*:
The lesson is that lambda* = dU*/d"exogenous limit." As such, it is a powerful shortcut—an alternative way to quickly find the responsiveness of the optimal value of the objective function as the exogenous limit in the constraint changes.

If, however, the constraint is relaxed a discrete amount, lambda* will not accurately measure the change in the maximum value function given the discrete change in the constraint.

As we have seen before in our discussions of the Method of Actual Comparison versus the Method of the Reduced Form, the question itself determines whether or not lambda* is applicable or not.

GETTING STARTED:

To complete this lab assignment, finish reading this document, then launch Excel and open C11Lab.xls.
Follow the instructions on the spreadsheet.

In addition to the lab, there is an exercise called C11Exer.xls which gives you an opportunity to apply your new Excel constrained maximization skills to a different problem.

A Comment on C11Exer.xls

C11Exer.xls marks a change in presentation. Unlike past workbooks, which have contained lots of pre-fabricated structure, C11Exer.xls is essentially blank. YOU have to create the structure yourself. There are no hints either.

Don't despair. Use what you're learned. Open previous labs, exercises, and answer keys. Look at how other problems we're set up and solved and shamelessly copy the format—that's the whole point of the Economic Approach. "To copy shamelessly?" No, that there is a fundamental framework that you can rely on and apply to any specific optimization problem.

One Parting Hint:

Since we hate throwing you out of the nest, we cannot resist one helpful suggestion. You will be asked to solve the C10Exer.pdf cards/books constrained optimization problem for varying sizes of shelf space.

It seems that many students are confused about how to use the ADD constraint in the Solver Parameters dialog box. The FIRST time you run Solver (with CBL=500), you must add the constraint that 0 = CBL - Brandy - 4*Cigar. After that, you never need to add a constraint again—you just change the value of CBL in the worksheet and rerun Solver.

For some reason, students seem to think that every time they run Solver, they must add the constraint again. That's just not so.

You should also be aware of the fact that you can remove unwanted constraints (and we SWEAR this is our last hint). Just click on the constraint you want to remove from the "Constraints" part of the Solver Parameters dialog box and then click on the "Delete" button.

Possible Bug:
Although Microsoft constantly updates and fixes its software, a common bug remains embedded in many older versions of Excel. If you have used a NAMED cell in a constraint, you may not be able to delete the constraint. An easy workaround if you experience this problem is to first remove the name (by accessing the Define Name dialog box by executing Insert: Name ...: Define and then deleting the name), then delete the constraint.

We hope you do not suffer too much separation anxiety as you begin to create your own workbooks. Remember to look at previous labs for inspiration as you work through the new challenges in C11Exer.xls! GOOD LUCK!!!