Chapter 7: Optimal Inventory Policy: A Single Variable Unconstrained Optimization Problem with Comparative Statics

SUGGESTED ANSWERS

Two of the five questions (numbers 2 and 3) involve pencil and paper calculations; and two of the questions involve using Excel (numbers 1 and 4).   Question 5 involves no calculations.

Part I: Solving the Optimal Inventory Problem

(1) Solve the minimize Total Inventory Cost problem in the concrete case with Q = 1,000, k = 64, a = 2000, b = 200. Do this using the Excel Solver.

See the file C7LabA.xls to see how to answer this question with Excel's Solver.

> The optimal number of TV sets to order per shipment is 250. This means you need four shipments (N*=4) and the average size of inventory is 125 TVs. D=250 will minimize the Total Inventory Cost at $216,000 for the company.

(2) On a separate sheet of paper, solve the retailer's problem (i.e., minimize Total Inventory Cost) using calculus in the *general* case. (By the general case, we mean using the exogenous variables a, b, Q and k instead of specific numerical values.) Show your work *neatly* on paper. Work your way through the five steps in the solution recipe. Take a look at Chapter 5 readings and Suggested Answers. Prominently display your answers for D* and TIC*. Put your name at the top.

STAGE I: Setting up the problem

What is the objective of the agent in words?

To minimize total inventory costs

What is the endogenous variable?

D, the size per shipment (or number of TVs in each shipment). As explained in detail in C7Lab.pdf, choice D determines number of shipments and average size of inventory, both of which are also endogenous variables.

What are the exogenous variables?

Q (number of TVs sold in a year), k (carrying charge in $ per TV), a (shipping charge in $ per shipment), and b (wholesale TV price in $ per TV)

STAGE II: Finding the Optimal Solution.

There are FIVE steps to solving the problem mathematically.

(1) Set up the problem mathematically; that is, write out the **objective function**, indicate the **choice variable**, and indicate what the agent wishes to do (i.e., **MAX** or **MIN**).

$$\min_{D} \ TIC \ = \ \frac{kD}{2} \ + \ \frac{aQ}{D} \ + \ bQ$$

(2) Take the **derivative** of the objective function with respect to the choice variable.

$$\frac{dTIC}{dD} \ = \ \frac{k}{2} \ - \ \frac{aQ}{D^2}$$

(3) Set the **derivative** of the objective function equal to **zero**. The value of the endogenous variable that satisfies this condition (called a "first-order condition" for you math types) is unique; we indicate this by sticking a star (*) next to it.

$$\frac{dTIC}{dD} \ = \ \frac{k}{2} \ - \ \frac{aQ}{D^{*2}} \ = \ 0$$

(4) Solve for the optimal value of the endogenous variable; that is, rearrange the above equation so that you have D* by itself on the left-hand side and only exogenous variables on the right. This is the **reduced form**: it tells you the optimal value of the endogenous variable for any set of values for the exogenous variables. You can plug in any values for Q, k, and a and out pops the optimal value of D.

$$\frac{k}{2} \ - \ \frac{aQ}{D^{*\,2}} \ = \ 0$$

$$\frac{k}{2} \ = \ \frac{aQ}{D^{*\,2}}$$

$$D^{*\,2} \ = \ \frac{2\,aQ}{k}$$

$$D^{*} \ = \ \sqrt{\frac{2\,aQ}{k}}$$

This expression for D* is a reduced form. It conveys how the optimal value of the endogenous variable depends upon the exogenous variables.

$$N^* = \frac{Q}{D^*}$$
$$= \frac{Q}{\sqrt{\dfrac{2aQ}{k}}}$$
$$= \sqrt{\frac{kQ}{2a}}.$$

(5) **Plug** the optimal value back into the objective function. This will give you what is called "the **maximum value function**". You can think of it as a sort of reduced form of the objective function: it gives the maximum value (or minimum value, in the case of, for example, the lifeguard problem) of the objective function as a function of the parameters of the problem. In this example, you can plug in any values for Q, a, and k into the expression for the maximum value function, and out pops the highest level of profits that are obtainable.

$$TIC^* = \frac{k\left[\sqrt{\dfrac{2\,aQ}{k}}\right]}{2} + \frac{aQ}{\left[\sqrt{\dfrac{2\,aQ}{k}}\right]} + bQ$$

Part II: Comparative Statics:

(3) Find the **reduced form** for D* from (2) and use it to find dD*/dk — the change in optimal order size given a change in carrying cost per TV. Put your answer on the paper which includes answers to question 2.

$$D^* = \sqrt{\frac{2\,aQ}{k}} = \sqrt{2\,aQ}\ \ k^{-\frac{1}{2}}$$

$$\frac{dD^*}{dk} = -\ \frac{1}{2}\sqrt{2\,aQ}\ \ k^{-\frac{3}{2}}$$

Evaluated at Q=1,000, a=2000, and k=64, dD*/dk = - 1.95. This says that when k increases by an infinitesimal amount, D* decreases by a 1.95-fold amount. The minus sign means the k and D* are inversely related—higher k means lower D*; while lower k causes higher D*.

Notice that we are using the derivative here merely to explore how the function changes. We are NOT interested in setting the derivative equal to zero and solving the resulting equation. We do that when we are FINDING THE OPTIMAL SOLUTION. That's NOT what we're doing now—we are now in COMPARATIVE STATICS. Always know where you are in the problem.

(4) Return to C7Lab.xls and **draw a graph** to illustrate how D* varies with a, the shipping cost.   Use Excel, including both Solver and the Comparative Statics Wizard, to do this as you vary the value of "a" from 2000 to 1000 in increments of 200. (NOTE: Enter – 200, negative 200, in the amount of shock dialog box.) Record your answer in Screen 3 of the Q&A worksheet (as per the instructions in the file).

See the file C7LabA.xls to see how to answer this question with Excel's Solver and the Comparative Statics Wizard.

The graph looks like this:
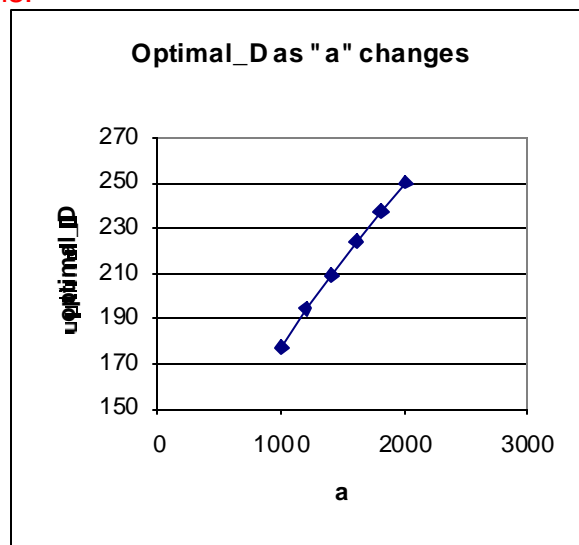
This graph says that as "a," the shipping cost, increases, the optimal size of the shipment increases. This makes sense, because a bigger shipment means FEWER shipments in a year. Notice, however, that the relationship is NOT constant. It looks like increases in shipping cost FROM A SMALL INITIAL VALUE OF SHIPPING COST

(at say $1000/shipment), has a GREATER effect than if the same change in shipping cost occurred FROM AN ALREADY HIGH VALUE OF SHIPPING COST (at say $2000/shipment).

(5) Use your answers to (3) and (4) above to suggest an explanation of the "JIT revolution" — that is, the observed reduction in inventory levels over the last few years. Do this in a text box in your Excel file labeled C7Lyour initials.xls.

Although this explanation is in a text box in the file C7LabA.xls, it is of such crucial importance that we repeat it here.

JIT is a change in observed behavior. Since economists view observed behavior as optimal behavior, they interpret JIT as a change in optimal behavior. To explain the JIT revolution, economists search for exogenous shocks that could have caused such a change.

Questions 3 and 4 were designed to get you to start thinking about the possible shocks that could move D*. JIT is nothing more than a low D* and high N*—many small shipments to meet sales needs. Such a move, from a high D* (low N*) yesterday to a low D* (high N*) today (and, hence, the "JIT revolution") could have been caused by a higher k and/or a lower "a." Has k, the carrying charge per shipment, risen? That's an empirical question. The econometric or testing side of economics would have us explore this question. Has "a," the shipping charge, fallen? Once again, empirical work (data gathering and analysis) is required. We note that trucking and airline deregulation in the late 1970s would certainly seem to have lowered shipping costs, and the price of gasoline since the high price era of the 1970s has fallen in real terms. (More recently, major improvements in information technology have made it possible for firms to keep better track of inventory. To capture this phenomenon would require us to alter our model.)

The point here, however, is that the Economic Approach "explains" JIT by searching for shocks in exogenous variables that would lead the agent to move from an initial optimal position of no JIT (high D* and low N*) to a new optimal position of JIT (low D* and high N*).

That interpretation of JIT is different from an often-heard non-economist's view that firms "got smarter." According to this argument, firms were making a mistake before and now they're "getting it right." We cannot overemphasize how markedly this differs from the economist's view that firm's ALWAYS OPTIMIZE. Economists assume that firms were optimizing before and they are optimizing now. If there's a change in an agent's observed behavior, economists' first resort is to look for a CHANGE IN THE ENVIRONMENT, not for a suddenly wiser agent. We can't say that this approach is the only valid way to look at the world, but it has proved a very fruitful way of understanding human behavior.